Dinámica Topológica

Equipo organizador

- Héctor Barge Yáñez (Universidad Politécnica de Madrid)
- Jaime Jorge Sánchez Gabites (Universidad Complutense de Madrid)

Descripción

La finalidad de esta sesión es ilustrar la variedad de líneas de investigación actuales que involucran aspectos topológicos de los sistemas dinámicos. Así, contiene charlas de temáticas muy variadas cuyo hilo conductor es que la topología juega un papel relevante, bien sea por la naturaleza de los resultados que se obtienen o de las técnicas que se utilizan para obtenerlos.

The purpose of this session is to illustrate the variety of current research lines that involve topological aspects of dynamical systems. The common thread of the talks is the presence of topology, either in the nature of the results that are discussed or as a tool to obtain them.

Palabras clave: Índice de Conley y de punto fijo, atractores, conjuntos de Julia, entropía, helicidad.

Programa

Lunes, 19 de enero

15:30 – 16:00	E. Vigil (Universidad de Oviedo) *Aplicaciones lineales a trozos 2-D: ruta hacia la coexistencia de atractores extraños*
16:00 – 16:30	L. Pardo (Universitat de Barcelona) *Transcendental Julia Sets as Cantor bouquets*
16:30 – 17:00	R. Cardona (Universitat de Barcelona) *On Arnold-Keshin's conjecture about flows with fixed helicity*

Martes, 20 de enero

11:00 – 11:30	E. Muñoz (Universidad Complutense de Madrid) *Index and bifurcation theory approach for the search of periodic solutions*
11:30 – 12:00	R. Barral (Universidad Politécnica de Madrid) *Mean equicontinuity, flow spaces and foliation theory*
12:00 – 12:30	P. Tópor (Gdansk University of Technology) *Fixed point indices of iterates of smooth boundary-preserving maps*
15:30 – 16:00	P. Oprocha (University of Ostrava) *On entropy in Lebesgue measure preserving maps*
16:00 – 16:30	P. Varandas (Universidade de Aveiro) *Generic homeomorphisms have maximal topological emergence*

Aplicaciones lineales a trozos 2-D: ruta hacia la coexistencia de atractores extraños

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Resumen. Las aplicaciones cuadráticas en dimensión uno son, por su complejidad dinámica, fuente de numerosos estudios. En este contexto, destaca especialmente la familia de *aplicaciones logísticas* $f_\mu(x) = \mu x(1-x)$. Un primer acercamiento al estudio de f_μ se realizó mediante aplicaciones lineales a trozos conocidas como *aplicaciones tienda*.
Si pasamos al plano, la complejidad de las aplicaciones cuadráticas es aún mayor. Siguiendo la idea desarrollada en dimensión uno, en [1] se introdujo una familia de aplicaciones lineales a trozos (*Expanding Baker Maps*) con la que se pretendía empezar a comprender y describir la dinámica de la familia de aplicaciones $T_{a,b} = (a + y^2, x + by)$.
El objetivo de esta charla es exponer las técnicas utilizadas en el estudio de las *Expanding Baker Maps* así como los propiedades obtenidas en los últimos años. Entre otros resultados, y quizá el más destacado, se demostró la coexistencia de atractores extraños bidimensionales mediante un proceso de renormalización ([2],[3]).

Referencias

[1] A. Pumariño, J. Á. Rodríguez, J. C. Tatjer, E. Vigil (2014). Expanding baker maps as models for the dynamics emerging from 3d-homoclinic bifurcations. *Discrete and Continuous Dynamical Systems - Series B*, 19(2), 523–541.

[2] A. Pumariño, J. Á. Rodríguez, E. Vigil (2018). Renormalization of two-dimensional piecewise linear maps: Abundance of 2-D strange attractors. *Discrete and Continuous Dynamical Systems - Series A*, 38(2), 941–966.

[3] A. Marqués-Lobeiras, A. Pumariño, J. Á. Rodríguez, E. Vigil (2023). Splitting and coexistence of 2-D strange attractors in a general family of Expanding Baker Maps. *Nonlinearity*, 36(8), 4247-4282.

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Transcendental Julia Sets as Cantor bouquets

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Resumen. In the study of the dynamics of transcendental entire functions, a central object of interest is the Julia set, which is the locus of chaotic behaviour under iteration. For many such functions, the Julia set consists of unbounded curves that escape to infinity and collectively form a Cantor bouquet: a topological structure ambiently homeomorphic to a straight brush. In this talk, we present examples of such phenomena and discuss criteria that ensure the Julia set takes this form. This is based on joint work with Lasse Rempe.

On Arnold-Keshin's conjecture about flows with fixed helicity

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Resumen. In the study of volume-preserving vector fields in three-dimensional manifolds, a topic tightly related to hydrodynamics, a very important notion is that of helicity. It is an invariant of these fields, with a topological interpretation, that exhibits some surprising uniqueness properties. Arnold and Khesin asked in 1992 whether certain sets of volume-preserving flows could be somewhere dense in the set of flows with a given value of helicity, conjecturing later that this might be the case for "some topology". After introducing this conjecture, we will show that it is false if the chosen topology is a very natural one in this context: the C^1-topology. This is based on joint work with Julian Chaidez (University of Southern California) and Francisco Torres de Lizaur (Universidad de Sevilla).

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Index and Bifurcation Theory approach for the search of periodic solutions

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Resumen. In this talk, we present two approaches to analysing the existence and multiplicity of periodic solutions in planar Hamiltonian periodic problems. Firstly, we characterise the Conley–Zehnder index in terms of the winding number in order to apply a suitable version of the Poincaré–Birkhoff theorem. Secondly, we adopt a less well-studied approach, analysing some eigenvalue algebraic multiplicities to establish local and global bifurcation results in the sense of Rabinowitz. The results presented in this talk have been obtained in collaboration with A. Boscaggin (University of Turin), J. C. Sampedro (UPM) and A. Tellini (UPM).

Mean equicontinuity, flows spaces and foliation theory.

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Resumen. The concept of mean equicontinuity is undoubtedly one of the most important areas of research in dynamical systems theory, lying at the intersection of topological and measure theory. In this talk, we will study this concept for flow spaces—that is, one-dimensional compact metric spaces admitting a fixed-point-free action of the real line—, where there is a strong correlation between topology and the possible dynamics. In order to do that, we will use equicontinuous laminations as topological substitutes for the role played by maximal equicontinuous factors in the category of group actions.

Fixed point indices of iterates of smooth boundary-preserving maps

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Resumen. Let us consider a smooth manifold M with a non-empty boundary ∂M. For a smooth map f that preserves ∂M and a fixed point $x_0 \in \partial M$, we determine all possible forms of the pairs of sequences of fixed point indices of iterates

$$(\mathrm{ind}(f^n, x_0), \mathrm{ind}(\bar{f}^n, x_0))_n,$$

where $\bar{f}$ denotes the restriction of f to ∂M. As an application of our result, we compute in some cases the Nielsen-type invariant $D_r(f; M, \partial M)$, which is equal to the minimal number of r-periodic points among all maps preserving ∂M and smoothly homotopic to f. This is a joint work with Grzegorz Graff and Jerzy Jezierski.

On entropy in Lebesgue measure preserving maps

Piotr Oprocha

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Resumen. The aim of this talk is to provide some insight into continuous maps of the interval which preserve the Lebesgue measure. Except for the identity map or $1 - id$ all such maps have topological entropy at least $\log 2/2$. Generically they have infinite topological entropy, are topologically mixing and Lebesgue measure is weakly mixing. In this talk we will present further topological properties of maps in this class and confront them with metric entropy. (Joint work with Jozef Bobok, Jernej Cinc and Serge Troubetzkoy)

Generic homeomorphisms have maximal topological emergence

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Resumen. Homeomorphisms on compact manifolds have a very complex dynamical behavior and, in particular, have infinite topological entropy. In this case, a refinement of the complexity can be measured through the topological emergence of the homeomorphism. In this talk I will discuss some recent results on the topological emergence of typical homeomorphisms, namely that the topological emergence of a C^0-generic homeomorphism is maximal, equal to the dimension of the manifold.