The failure of Weyl and Levi's Theorems in the compatible Lie algebra setting

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Resumen.
Two of the most fundamental results in the representation theory of Lie algebras are Weyl's Theorem and Levi's Theorem (valid for fields of characteristic zero). Weyl's Theorem tells us that finite-dimensional representations of simple Lie algebras are always decomposable into irreducible representations, which are much simpler to understand. Levi's Theorem allows us to decompose any Lie algebra into a semidirect product of a solvable ideal and a semisimple subalgebra, whose representation theory is well-known and simpler to work with than the general case.
A related notion to Lie algebras is that of *compatible Lie algebras*: given two Lie products on a vector space, they are said to be compatible if any linear combination of them is also a Lie product. These algebras arose from the related class of compatible Poisson algebras in the context of mathematical physics and Hamiltonian mechanics.
Despite many similarities with Lie algebras, compatible Lie algebras are sometimes strangely behaved. In this talk we explore how the aforementioned Weyl and Levi's theorems fail in this setting and discuss the implications for the development of a theory for compatible Lie algebras.

Palabras clave: Compatible Lie Algebras; Representation theory; Weyl Theorem; Levi Theorem.

References

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