Reduced order methods for parametric time-dependent partial differential equations

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Resumen. Reduced-order methods can significantly reduce the computational cost required to obtain numerical approximations while also trying to provide sufficiently accurate approximations. A frequent case is that of equations that depend on one or several parameters. The challenge in this case is to provide accurate approximations for parameter values that are no part of the dataset used to compute the reduced-order basis. In this talk, we consider a type of reduced order methods that are called proper orthogonal decomposition (POD) methods.
In the present talk we propose a new POD method for parametric time dependent reaction-diffusion partial differential equations. The method is based on finite differences (respect to time and parameters) of some snapshots. The snapshots are finite element approximations evaluated at different times and values of parameters in a selected set. The method is designed in such a way that pointwise in time estimates can be proved at any time in a given interval. The a priori bounds are also valid for any value of the parameters (including out of sample values). The error in the POD approximations depend on the tail of the eigenvalues (sum of nonzero eigenvalues of the correlation matrix of the data set in the POD method, starting from a selected one) and on the distance between two consecutive values of time where the snapshots are taken and the distance between two consecutive values of the parameters. We also consider a standard POD method based on the set of snapshots for different values of time and parameters and prove analogous error bounds using the techniques developed for the new method. For the standard case the bounds are quasi-optimal, as the exponent in the tail of the eigenvalues depends on the smoothness of the finite element approximations. As smoother the approximations are, the closer the exponent gets to the optimal value one.
Some numerical experiments show the performance of both new and tradicional methods.

Palabras clave: POD-ROM methods; parametric equations; reaction-diffusion equations; pointwise estimates in time.

Referencias

[1] B. García-Archilla, V. John, J. Novo (2025). POD-ROM methods: from a finite set of snapshots to continuous-in-time approximations *SIAM J. Numer. Anal.*, 63, 800–826.

[2] B. García-Archilla, A. García-Mascaraque, J. Novo (2025). POD-ROM methods: error analysis for continuous parametrized approximations (enviado).

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