



## On Hilbert's 19th problem on regularity for elliptic equations and recent extensions

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**Resumen.** In 1900 Hilbert's 19th problem asked whether minimizers of elliptic functionals are always analytic. But in fact, at that time, the existence of minimizers was not yet understood, even for the Dirichlet functional associated to the Laplacian. As pointed out in [1], it was Beppo Levi who demonstrated its existence in 1906 (with an extraordinary argument that we will review), giving rise to the "Riesz-Fréchet theorem" of 1907. We will continue with the progress made on the 19th Hilbert problem by explaining the celebrated result of De Giorgi and Nash in the late 1950s. We will then turn into te regularity of stable solutions, a larger class than absolute minimizers. We will mention very recent results establishing long-standing conjectures: first for minimal surfaces (starting from the 2021 work of Chodosh and Li) and then for reaction-diffusion equations (describing our result [2]). We will finally mention more recent semilinear extensions of [2] and make a list of open problems.

**Palabras clave:** Regularity for nonlinear elliptic PDEs; Stable solutions; Semilinear elliptic equations; Minimal surfaces.

## Referencias

- [1] H. Brezis, F. Browder (1998). Partial differential equations in the 20th century. Adv. Math., 135, 7–144.
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