Endpoint estimates for the fractal circular maximal function and related local smoothing

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Resumen. The spherical maximal function is the operator defined by

$$\sup_{t>0}\Big|\int_{\mathbb{S}^{d-1}} f(x-ty)d\sigma(y)\Big| \quad x\in\mathbb{R}^d,$$

where $d\sigma$ is the normalized surface measure on the unit sphere $\mathbb{S}^{d-1}$. E. M. Stein (for $d\geq 3$) and J. Bourgain (for $d=2$) proved that this operator is bounded on $L^p(\mathbb{R}^d)$ if and only if $p>d/(d-1)$. The spherical maximal function is a relevant object in harmonic analysis, connected to the solutions of the wave equation and related smoothing properties, and variants of it have been widely studied in the literature. In recent times, there has been an increasing interest in understanding sharp forms of L^p–L^q estimates for the spherical maximal function when the supremum is taken over dilation sets of fractal dimensions of different nature. In this talk we will give a general overview of the topic, and present our contributions. More particularly, we will prove missing endpoint estimates for the fractal spherical maximal function which were open when $d=2$, and study closely related L^p–L^q local smoothing estimates for the wave operator over fractal dilation sets. Our approach relies on bilinear restriction estimates for the cone due to T. Wolff and T. Tao.
This is joint work with Sanghyuk Lee, Feng Zhang, and Shuijiang Zhao.

Palabras clave: Fractal set; circular maximal function; local smoothing.

Agradecimientos. The author is supported by the Basque Government through the BERC 2022-2025 program and by the Spanish Ministry of Science and Innovation: BCAM Severo Ochoa accreditation CEX2021-001142-S/MICIN/AEI/10.13039/501100011033, PID2023-146646NB-I00 funded by MICIU/AEI/10.13039/501100011033 and by ESF+ and CNS2023-143893, and by IKERBASQUE