Advances in Linear Algebra, Matrix Analysis and their Applications

Equipo organizador

- Álvaro Samperio Valdivieso (CUNEF Universidad)
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Descripción

Las ponencias de esta sesión, expuestas por miembros de la Red de Excelencia ALAMA, ofrecerán un amplio abanico de temas clásicos y recientes cuyo hilo conductor es el álgebra lineal y el análisis matricial; en concreto, se abarcarán áreas tales como la teoría de matrices (factorización de matrices, matrices estructuradas, matrices totalmente positivas. . .), matrices polinomiales y racionales, matrices aleatorias, caminos aleatorios en redes, teoría de control, sistemas lineales y sus soluciones, retículos, la teoría espectral de matrices y problemas inversos de autovalores. El conjunto de estas comunicaciones tendrá un doble objetivo: por un lado, presentar los avances recientes en resultados fundamentales y, por otro lado, mostrar diversas aplicaciones, tales como la regresión polinomial -en estadística- o el desarrollo de métodos numéricos (entre otros, para la integración y evaluación de funciones) precisos y estables.

Palabras clave: teoría de control lineal; teoría espectral de matrices; subespacios; algoritmos numéricos; caminos aleatorios.

Programa

Jueves, 22 de enero

11:00 – 11:30	Miguel Carriegos (Universidad de León) *Álgebra lineal en la categoría de las acciones de realimentación*
11:30 – 12:00	Aida Abiad (Eindhoven University of Technology) *A spectral approach to Kemeny's constant*
12:00 – 12:30	Alicia Roca (Universitat Politècnica de València) *On row completion of polynomial and rational matrices and partial prescription of their eigenstructures*
12:30 – 13:00	M. Eulàlia Montoro (Universidad de Barcelona) *Endomorfismos con subespacios hiperinvariantes isomorfos*
15:30 – 16:00	Carlos Marijuán (Universidad de Valladolid) *The INIEP*
16:00 – 16:30	Julio Moro (Universidad Carlos III de Madrid) *Compensation of signs in the Symmetric Nonnegative Inverse Eigenvalue Problem*
16:30 – 17:00	Francisco Enrique Velasco (UPV/EHU) *Matriz más próxima con un valor propio prescrito con característica de Weyr mayorizando a la partición* $(2,1)$
17:00 – 17:30	Laureano González-Vega (CUNEF Universidad) *Characterising the eigenvalues of* QM*-matrices and* $Q^{1,2}$*-matrices*
18:00 – 18:30	Àlvar Martín (Universitat Politècnica de Catalunya) *Green Matrices in Schrödinger-Type Network*
18:30 – 19:00	Begoña Cantó (Universitat Politècnica de València) *The study of the combined matrices*

Viernes, 23 de enero

11:00 – 11:30	José-Javier Martínez (Universidad de Alcalá de Henares) *Accurate computation of the hat matrix in polynomial regression*
11:30 – 12:00	Esmeralda Mainar (Universidad de Zaragoza) *Stability and Accuracy in Bernstein Representations*
12:00 – 12:30	Ujué Etayo (CUNEF Universidad) *Random matrices and optimal point configurations*

Álgebra lineal en la categoría de las acciones de realimentación

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Resumen. El estudio matemático de los sistemas de control se remonta a 1868 con unos primeros trabajos de Maxwell [5] sobre el control del regulador de bolas para la máquina de vapor donde se introduce la noción de realimentación. El año 1943 nace la Cibernética como disciplina científica que unifica la realimentación como herramienta fundamental desde control automático de sistemas hasta la actividad del sistema nervioso humano, los métodos de comportamiento y de aprendizaje [1, 6, 7]. Los sistemas dinámicos lineales constituyen la parte más básica y, a la vez, más efectiva de la teoría de control. Brunovsky [3] y Kalman clasifican los sistemas lineales sobre espacios vectoriales en 1970, pero el problema general de clasificación de sistemas lineales es *wild* [2]. No obstante, el problema de clasificación de sistemas lineales sobre R-módulos se resuelve en 2013 [4] para cualquier anillo conmutativo R en la clase de sistemas *regulares*.

En esta charla revisaremos la noción de morfismo en la categoría de sistemas lineales y acciones de realimentación como la adecuada generalización de las acciones de realimentación, que resultarán ser los isomorfismos de la categoría. También introduciremos la maquinaria de álgebra lineal necesaria para estudiar los morfismos de realimentación: núcleos, conúcleos y estructura exacta en la categoría de sistemas lineales. Finalmente revisaremos los resultados clásicos de descomposición de sistemas. En particular encontraremos los objetos simples en la categoría de sistemas lineales y probaremos que la categoría de sistemas lineales es semisimple en objetos.

Referencias

[1] M. Arbib (1987). *Brains Machines and Mathematics.* Springer-Verlag.

[2] J. W. Brewer, L. Klingler (2001). On feedback invariants for linear dynamical systems. *Linear Algebra Appl.*, 325, 209–220.

[3] P. A. Brunovsky (1970). A classification of linear controllable systems. *Kybernetika* 3, 173–187.

[4] M. V. Carriegos (2013). Enumeration of classes of linear systems via equations and via partitions in a ordered abelian monoid. *Linear Algebra Appl.*, 438, 1132–1148.

[5] J. C. Maxwell (1868). On Governors. *Proc. Roy. Soc.* 16, 270–283.

[6] W. S. McCulloch, W. H. Pitts (1943). A logical calculus of the ideas immanent in nervous activity. *Bull. Math. Biophys.* 5, 115–133.

[7] A. Rosenblueth, N. Wiener and J. Bigelow (1943). Behavior, Purpose and Teleology. *Philos. Sci.* 10, 18–24.

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A spectral approach to Kemeny's constant

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Resumen. Kemeny's constant, a fundamental parameter in the theory of Markov chains, has recently received significant attention within the graph theory community. Originally defined for a discrete, finite, time-homogeneous, and irreducible Markov chain based on its stationary vector and mean first passage times, Kemeny's constant finds special relevance in the study of random walks on graphs. Kemeny's constant gives a measure of how quickly a random walker can move around a graph and is thus a good measure of the connectivity of a graph. Kemeny's constant has many useful interpretations, including the spread of infectious diseases (how quickly a disease will reach epidemic levels), molecular conformation dynamics (presence or absence of metastable sets), and urban road networks (how well connected a network is). In general, a lower Kemeny's constant means that a graph is more connected, and a higher Kemeny's constant means that a graph is less connected. For these and other applications, the main question is: how do changes in the network lead to changes in Kemeny's constant? In this talk we investigate the effect of the network structure on Kemeny's constant. We do so by showing several new approximations for Kemeny's constant, which we derive using spectral graph theory techniques.

On row completion of polynomial and rational matrices and partial prescription of their eigenstructures

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Resumen. We solve the problem of characterizing the existence of a polynomial or a rational matrix when its complete structural data and some of its rows are prescribed. The complete structural data of a rational matrix is formed by the invariant rational functions, the invariants orders at infinity, and the column and row minimal indices.

We have also solved the same problem under partial prescription of the structural data, i.e., we have characterized the existence of a polynomial or a rational matrix when only some of the four types of invariants are prescribed leaving some freedom for the rest of them. The problem have been solved completely, analyzing the 15 different cases of partial prescription. We show here some examples.

Referencias

[1] A. Amparan, I. Baragaña, S. Marcaida, A. Roca (2024). Row or column completion of polynomial matrices of given degree. *SIAM J. Matrix Anal. Appl.*, 45(1), 478–503.

[2] A. Amparan, I. Baragaña, S. Marcaida, A. Roca (2025). Row or column completion of polynomial matrices of given degree II. *Linear Algebra Appl.*, 798, 252–279.

[3] A. Amparan, I. Baragaña, S. Marcaida, A. Roca (2025). Row completion of polynomial and rational matrices. *Linear Algebra Appl.*, 720, 109-138.

[4] A. Amparan, I. Baragaña, S. Marcaida, A. Roca (2025). Row completion of polynomial and rational matrices II. Preprint.

Endomorfismos con subespacios hiperinvariantes isomorfos

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Resumen. Sea $\mathbb{F}$ un cuerpo arbitrario, sea $f \in \mathrm{End}(\mathbb{F}^n)$ un endomorfismo. Tres retículos de subespacios vectoriales se asocian de forma natural al endomorfismo f: el retículo de los subespacios invariantes ([1], [3]), el de los subespacios característicos ([4]) y el de los subespacions hiperinvariantes ([2]).

Un subespacio vectorial $V \subseteq \mathbb{F}^n$ es *invariante* respecto a $f \in \mathrm{End}(\mathbb{F}^n)$ si $f(V) \subset V$, es *hiperinvariante* si es invariante para todo $g \in \mathrm{End}(\mathbb{F}^n)$ tal que $gf = fg$. Un subespacio invariante es *característico* si es invariante para todo $g \in \mathrm{Aut}(\mathbb{F}^n)$ tal que $gf = fg$.

Una caracterización de los isomorfismos entre retículos de subespacios invariantes puede encontrarse en [1]; los isomorfismos entre retículos de subespacios característicos son actualmente objeto de estudio. En este trabajo presentamos una caracterización de los isomorfismos entre retículos de subespacios hiperinvariantes ([5]) (ver también [6]). El propósito de esta charla es mostrar algunas ideas sobre esta última caracterización y algunas de las técnicas usadas en su resolución.

Referencias

[1] L. Brickman, P. A. Fillmore (1967). The invariant subspace lattice of a linear transformation. *Can. J. Math.*, 19, 35, 810–822.

[2] P. A. Fillmore, D. A. Herrero and W. E. Longstaff (1977). The hyperinvariant subspace lattice of a linear transformation. *Linear Algebra Appl.* 17, 125–132.

[3] I. Gohberg, P. Lancaster, L. Rodman (1986). *Invariant Subspaces of Matrices with Applications.* SIAM.

[4] D. Mingueza, M. E. Montoro, A. Roca (2018). The lattice of characteristic subspaces of an endomorphism with Jordan-Chevalley decomposition. *Linear Algebra Appl.*, 558, 63-73.

[5] D. Mingueza, M. E. Montoro, A. Roca (2024). Isomorphisms between lattices of hyperinvariant subspaces. *Linear Algebra Appl.*, 703, 395–422.

[6] P. Y. Wu (1992). Which linear transformations have isomorphic hyperinvariant subspace lattices? *Linear Algebra Appl.*, 169, 163–178.

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The INIEP

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Resumen. A list Λ of n complex numbers is said to be *realizable* if there exists an n-by-n nonnegative matrix whose spectrum is Λ. The problem of characterizing all realizable lists Λ is the *Nonnegative Inverse Eigenvalue Problem* (NIEP), see [2]. Another approach to the NIEP is to view a nonnegative matrix as the adjacency matrix of a weighted digraph and focus the attention on the coefficients of its characteristic polynomial $P(x)$, see [1, 4]. The polynomial $P(x)$ is said to be *realizable* if there is a weighted digraph (equivalently an entry-wise nonnegative matrix) whose characteristic polynomial is $P(x)$.
An n-by-n matrix is *irreducible* if it is not similar by a permutation matrix to a nontrivially block triangular matrix. A digraph is strongly connected if every two vertices are joined by a path. Strongly connected digraphs are exactly the ones that have an irreducible adjacency matrix. Our interest here lies in identifying, among realizable spectra/polynomials, those that are realizable by an irreducible matrix/a strongly connected weighted digraph. This problem is the *irreducible* NIEP (INIEP).
After giving some general background, we make some useful new observations and show the existence of irreducible nonnegative realizations in some general cases. Then, we focus on $n < 5$, where the NIEP is solved. Finally, we focus on the trace 0 case and, using graph theoretic methods, characterize nonnegative irreducible realizability among realizable polynomials, [3].

Referencias

[1] D. M. Cvetković, M. Doob, H. Sachs (1995) *Spectra of Graphs.* Johann Ambrosius Barth.

[2] C. R. Johnson, C. Marijuán, P. Papparella, M. Pisonero (2018). The NIEP. *Operator theory, operator algebras, and matrix theory.* Birkhäuser/Springer, 353-372.

[3] C. R. Johnson, C. Marijuán, M. Pisonero (2025). Irreducible Nonnegative Realizations of Certain Spectra (submitted).

[4] J. Torre-Mayo, M. R. Abril-Raymundo, E. Alarcia-Estévez, C. Marijuán, M. Pisonero (2007). The nonnegative inverse eigenvalue problem from the coefficients of the characteristic polynomial. EBL digraphs. *Linear Algebra Appl.* 426, 729-773.

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Compensation of signs in the Symmetric Nonnegative Inverse Eigenvalue Problem

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Resumen. C-realizability was originally introduced in [1] as a sufficient condition for the Real Nonnegative Inverse Eigenvalue problem (RNIEP)[a].It was shown back then that C-realizability included as particular cases most of the known sufficient conditions for the RNIEP.

It was not until 2017 that it was shown in [2] that C-realizability was more closely related to the Symmetric Nonnegative Inverse Eigenvalue problem (SNIEP))[b] than to the RNIEP.

The combinatorial nature of the original definition of C-realizability has conduced over the years to combinatorial characterizations of the set of C-realizable lists. In this talk we will first review a partial one, obtained in [4] for real lists with zero sum, and then the most general combinatorial characterization, obtained in [5] for arbitrary lists of real numbers. One of the most remarkable consequences of the latter characterization is that it proves the monotonicity of C-realizability, i.e., that the operation of increasing any positive entry of a C-realizable list preserves C-realizability.

[a] The RNIEP consists, for a given positive integer n, in characterizing those lists of n real numbers which are the spectrum of some $n \times n$ matrix with real entries.

[b] The SNIEP consists, for a given positive integer n, in characterizing those lists of n real numbers which are the spectrum of some $n \times n$ real symmetric matrix. It is known since 1996, see [3], that the two problems are different.

Referencias

[1] A. Borobia, J. Moro, R. L. Soto (2008). A unified view on compensation criteria in the real nonnegative inverse eigenvalue problem. *Linear Algebra Appl.*, 428, 2574–2584.

[2] R. Ellard & H. Smigoč (2017). Connecting sufficient conditions for the Symmetric Nonnegative Inverse Eigenvalue Problem. *Linear Algebra Appl*, 498, 521–552.

[3] C. R. Johnson, T. J. Laffey & R. Loewy (1996). The Real and the Symmetric Nonnegative Inverse Eigenvalue Problem are different. *Proc. Am. Math. Soc.*, 124, 3647–3651.

[4] C. Marijuán & J. Moro. (2021) A characterization of trace-zero sets realizable by compensation in the SNIEP.*Linear Algebra Appl.*, 615, 42–76.

[5] C. Marijuán & J. Moro. (2024) A characterization of sets realizable by compensation in the SNIEP.*Linear Algebra Appl.*, 693, 425–447.

Matriz más próxima con un valor propio prescrito con característica de Weyr mayorizando a la partición $(2, 1)$

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Resumen. Sea $A \in \mathbb{C}^{n\times n}$ y $z_0 \in \mathbb{C}$. Denotemos por $\|\cdot\|$ a la norma espectral. Tratamos de determinar la distancia de A al conjunto de matrices Z que tienen a z_0 como valor propio cuya partición, $\mathrm{w}(z_0, Z)$, en la característica de Weyr de Z mayoriza a $(2, 1)$. Concretamente, pretendemos probar la igualdad

$$\min_{\substack{Z\in\mathbb{C}^{n\times n}\\ (2,1)\prec\!\!\prec \mathrm{w}(z_0,Z)}} \|Z - A\| = \max_{t\geqslant 0} \sigma_{2n-2}\left(\begin{bmatrix} z_0 I_n - A & tI_n \\ O & z_0 I_n - A \end{bmatrix}\right).$$

Referencias

[1] G. Armentia, J. M. Gracia, F. E. Velasco (2020). Nearest matrix with a prescribed eigenvalue of bounded multiplicities. *Linear Algebra Appl.*, 592, 188-209.

[2] Kh. D. Ikramov, A. M. Nazari (2003). On the distance to the closest matrix with triple zero eigenvalue. *Mathematical Notes*, 73 no. 4, 511–520.

[3] A. N. Malyshev (1999). A formula for the 2-norm distance from a matrix to the set of matrices with multiple eigenvalues. *Numer. Math.*, 83 no. 3, 443–454.

[4] E. Mengi (2011). Locating a nearest matrix with an eigenvalue of prespecified algebraic multiplicity. *Numer. Math.*, 118, 109–135.

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Characterising the eigenvalues of QM-matrices and $Q^{1,2}$-matrices

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Resumen. P-matrices are matrices all of whose principal minors are positive. Q-matrices are matrices whose sums of principal minors of the same order are positive. A matrix is a PM-matrix if all its powers are P-matrices. A matrix is a QM-matrix if all its powers are Q-matrices. The study of the eigenvalues of these matrices (and its powers) brings many open questions. For example it was not known until 2024 that the eigenvalues of a PM-matrix are necessarily positive or it is not known if the eigenvalues of a matrix A such that A and A^2 are P-matrices necessarily have positive real parts. In order to study these two questions, we will characterise the real QM-matrices up-to size 4 and we characterise those real matrices A, 4×4, such that A and A^2 are Q-matrices but not all eigenvalues of A have positive real part.

Green Matrices in Schrödinger-Type Network

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Resumen. This paper introduces and studies a generalization of classical random walks on finite networks, called Schrödinger random walks. Unlike classical random walks, this model allows for nonzero stay probabilities at nodes and incorporates node-specific weights, as well as a potential parameter $\lambda \geq 0$.

The authors begin by introducing a generalized transition matrix in which transition probabilities are based on a conductance function and a positive weight function $\boldsymbol{\omega}$, leading to a new transition matrix $\mathsf{P}_{\lambda,\boldsymbol{\omega}}$ that depends on both λ and $\boldsymbol{\omega}$. Building on this, the authors define a Schrödinger-type operator $\mathsf{L}_{\lambda,\boldsymbol{\omega}}$, which is symmetric, positive semidefinite, and becomes singular when $\lambda = 0$.

The study then focuses on generalized inverses of the matrix $\mathsf{F}_{\lambda,\boldsymbol{\omega}} = \mathsf{L}_{\lambda,\boldsymbol{\omega}} - \lambda\boldsymbol{\omega}\boldsymbol{\omega}^T$. Among these, particular attention is given to its group inverse, which plays a central role in solving the Poisson equation and in characterizing the dynamics of the random walk. Using this framework, the authors derive the Mean First Passage Times matrix (MFPT) in terms of the generalized inverses, and the classical notion of Kemeny constant.

Finally, we introduce the concept of equilibrium measure and its associated capacity. These allow for the construction of an equilibrium matrix, which is shown to be a valid Green matrix. This perspective offers an alternative and physically meaningful way to express both the MFPT and Kemeny's constant.

Referencias

[1] Á. Carmona, M. J. Jiménez, À. Martín (2023). Mean first passage time and Kemeny's constant using generalized inverses of the combinatorial Laplacian. *Linear Multilinear A.*, 1–15.

[2] Á. Carmona, A. M. Encinas, M. J. Jiménez, À. Martín (2023). Random Walks associated with symmetric M-matrices. *Linear Algebra Appl.*, 693, 324–338.

[3] Á. Carmona, A. M. Encinas, M. J. Jiménez, À. Martín. Network Parameters via Equilibrium Measures in Schrödinger Random Walks. Submitted.

[4] E. Bendito, Á. Carmona, A. M. Encinas, J. M. Gesto (2010). Characterization of symmetric M-matrices as resistive inverses. *Linear Algebra Appl.*, 430, 1336–1349.

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The study of the combined matrices

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Resumen. A combined matrix is defined as a square matrix $\mathcal{C}(A) = A \circ A^{-T}$, where $A = (a_{ij})$ is a nonsingular matrix and $\circ$ denotes the Hadamard (entrywise) product. In control theory, this matrix is known as the Relative Gain Array (RGA) matrix [1], and it plays a key role in selecting optimal input–output pairings in multivariable process control systems. Toeplitz matrices, characterized by constant entries along each diagonal, are an important class of structured matrices and have been widely studied for their algebraic and computational properties (see, for example, [2]).
In this work, we explore the structure of the combined matrix $\mathcal{C}(A)$ when the original matrix A is a Toeplitz matrix with a prescribed tridiagonal structure. We focus on a perturbed family of such matrices, where the classical tridiagonal Toeplitz form is modified by introducing a scalar factor k in the $(1, 2)$ entry. Specifically, we consider matrices of the form

$$A_n^{(12)} = T^{(12)}(n; c, a, b) = \begin{pmatrix} a & kb & 0 & \cdots & 0 & 0 & 0 \\ c & a & b & \cdots & 0 & 0 & 0 \\ 0 & c & a & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & c & a & b \\ 0 & 0 & 0 & \cdots & 0 & c & a \end{pmatrix}, \qquad k \notin \{0, 1\}.$$

This extends the study initiated in [3], where the combined matrices of classical tridiagonal Toeplitz matrices were shown to be bisymmetric and doubly quasi-stochastic under the condition $bc = a^2$. Here, we prove that when $bc = a^2$, the combined matrix $\mathcal{C}(A_n^{(12)})$ always exists despite the perturbation, and we provide its explicit analytical form.

Referencias

[1] E. Bristol (1966). On a new measure of interaction for multivariable process control. *IEEE Trans Automat Contr.*, 11, 133–134.

[2] A. M. Encinas, M. J. Jiménez (2018). Explicit inverse of a tridiagonal (p,r)-Toeplitz matrix. *Linear Algebra Appl.*, 542, 402–421.

[3] B. Cantó, R. Cantó, A. M. Urbano (2025). Combined Matrix of a Tridiagonal Toeplitz Matrix. *AXIOMS*, 14 (375), 1–21.

Accurate computation of the hat matrix in polynomial regression

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Resumen. This talk deals with an example of the application of numerical linear algebra to statistics: the computation of the *hat matrix* (the projection matrix) in polynomial regression. A main tool is the use of the *QR factorization*, presented by G. H. Golub in [1]. In connection with this issue, it is interesting to recall the sentence by Golub included in [2]: *So I tried to get the statisticians interested in doing numerical computations. A few people were interested in it, but I don't think it had a heavy influence in statistics. I doubt if today people really use decomposition methods rather than normal equations. Statisticians are fairly fixed in their ways.*
Here we will mainly consider the accurate computation of the hat matrix (see [3]), whose role in statistics is analyzed in detail in [6]. The use of the Lagrange basis for polynomial least squares fitting has recently been addressed in [5], but now we will be using the *monomial basis*, which leads to the use of *Vandermonde matrices*. When using the monomial basis and nonnegative nodes ordered as $0 \leq x_1 < \ldots < x_n$, then the corresponding (rectangular) Vandermonde matrix is *totally nonnegative*, and for this class of matrices P. Koev has introduced in [4] an algorithm to compute the QR factorization starting from the bidiagonal decomposition of the corresponding matrix.
We will show how to apply these tools to accurately compute the hat matrix H in polynomial regression problems. Also, from the point of view of computational complexity it is interesting to observe (see subsection 10,2 of [6]) that in certain applications only the diagonal entries of H are needed.

Referencias

[1] G. H. Golub (1965). Numerical Methods for Solving Linear Least Squares Problems. *Numerische Mathematik*, 7, 206–216.

[2] N. H. Higham (2008). An interview with Gene Golub.*MIMS Eprint 2008.8.* Manchester Institute for Mathematical Sciences, Univ. of Manchester.

[3] D. C. Hoaglin, R. E. Welsch (1978). The hat matrix in regression and ANOVA. *The American Statistician*, 32(1), 17–22.

[4] P. Koev (2007). Accurate computations with totally nonnegative matrices. *SIAM J. Matrix Anal. Appl.*, 29, 731–751.

[5] A. Marco, J. J. Martínez, R. Viaña (2024). Total positivity and least squares problems in the Lagrange basis. *Numer. Linear Algebra Appl. 2024*, e2554.

[6] G. A. F. Seber, A. J. Lee (2003) *Linear Regression Analysis (2nd edition).* Wiley Series in Probability and Statistics. Wiley-Interscience, Hoboken, NJ.

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Stability and Accuracy in Bernstein Representations

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Resumen. Building on foundational insights into the accuracy and stability of numerical algorithms (cf. [1]), this talk offers a comprehensive overview of recent backward and forward error analyses for corner-cutting algorithms used in the evaluation of functions expressed in Bernstein and Bernstein-related bases, both univariate and multivariate. We revisit key results on the conditioning of these bases, highlighting their implications for numerical stability.
The talk also presents recent progress in the development of algorithms that exploit the structure of totally positive matrices. These methods enable the accurate and efficient solution of interpolation and approximation problems in Bernstein-type bases, achieving high relative accuracy even in challenging computational settings.

Referencias

[1] N. J. Higham, (2002). *Accuracy and Stability of Numerical Algorithms: Second Edition.* SIAM.

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Random matrices and optimal point configurations

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Resumen. We begin with the following problem: we aim to identify families of points (i.e., sequences of sets of points) in 2-point homogeneous spaces for use in integration processes via the Monte Carlo method. One way to assess the quality of a family of points is by computing its discrepancy: families with low discrepancy constitute suitable integration nodes. In this work, we propose the use of point families derived from random matrices and compute the discrepancy associated with such families.

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