Análisis Complejo y Teoría de Operadores

Equipo organizador

- Tanausú Aguilar Hernández (Universidad de Málaga)
- Adrián Llinares Romero (Universidad Autónoma de Madrid)
- Alejandro Mas Mas (Universidad de Alicante)
- Elena de la Rosa Pérez (Universidad de Málaga)

Descripción

Tal y como su título indica, esta propuesta se enmarca dentro del Análisis Complejo y la Teoría de Operadores, disciplinas que ocupan sendos lugares relevantes dentro del Análisis Matemático y están estrechamente relacionadas con otras importantes como bien puedan ser el Análisis Armónico, la Geometría, las Ecuaciones en Derivadas Parciales o la Teoría de Números. El objetivo principal de la sesión es el de reunir a una selección de miembros de la Red *Análisis Matemático y Aplicaciones* (constituida en 2023 a partir de la unión de las redes temáticas *Análisis Funcional y Aplicaciones* y *Variable Compleja, espacios de funciones y operadores entre ellos*, ambas bien consolidadas) cuya investigación encaje en el área para así poder compartir ideas y promover la colaboración entre los diferentes nodos de la Red. Esta sesión se ha realizado en todos los Congresos Bienales de la RSME, a excepción de la edición de 2024. La selección de ponentes incluye tanto a investigadores establecidos como a doctores recientes y estudiantes con tesis en curso.

Palabras clave: Dinámica Compleja; Operadores de tipo Volterra y Toeplitz; Subespacios Invariantes y Ciclicidad; Derivadas Schwarzianas; Operadores de Composición (Ponderados).

Programa

LUNES, 19 de enero

15:30 – 16:00	Eva Gallardo (Universidad Complutense de Madrid) *Insights on the Invariant Subspace Problem*
16:00 – 16:30	Antoni López-Martínez (Universitat Politècnica de València) *The solution to the* $T \oplus T$*-recurrence problem*
16:30 – 17:00	Alberto Dayan (Universitat Autònoma de Barcelona) *Cyclicity of singular inner functions on Besov and coefficient spaces*
17:00 – 17:30	María J. Martín (Universidad de La Laguna) *Higher order harmonic Schwarzian derivatives*

MARTES, 20 de enero

11:00 – 11:30	Luis Rodríguez-Piazza (Universidad de Sevilla) *Some questions about weighted Hardy spaces*
11:30 – 12:00	Jesús Oliva-Maza (Universidad de Zaragoza) *Surjectivity of the translated weighted composition operators*
12:00 – 12:30	Manuel D. Contreras (Universidad de Sevilla) *Boundary behavior of the iterates of a holomorphic self-map of the unit disc*

15:30 – 16:00	Francisco J. Cruz-Zamorano (Universidad de Sevilla) *The rate of convergence of orbits in complex dynamics*
16:00 – 16:30	Oscar Blasco (Universidad de Valencia) *Cèsaro-type operators on mixed norm spaces*
16:30 – 17:00	Daniel Girela (Universidad de Málaga) *Rhaly operators acting on spaces of analytic functions*
17:00 – 17:30	Carme Cascante Canut (Universitat de Barcelona) *Words of analytic paraproducts on Bergman spaces induced by smooth rapidly decreasing weights*

18:00 – 18:30	Álvaro Miguel Moreno López (Universidad de Málaga) *Fractional Volterra-type operator*
18:30 – 19:00	Bozhidar E. Mihaylov (Universidad Autónoma de Madrid) *On the range of Toeplitz operators*

Insights on the Invariant Subspace Problem

Eva A. Gallardo-Gutiérrez

Universidad Complutense de Madrid & Instituto de Ciencias Matemáticas ICMAT

egallard@ucm.es

Resumen. The Invariant Subspace Problem, dating back to von Neumann's works in 1950, remains one of the central open problems in Operator Theory on Hilbert spaces, inspiring deep connections with areas such as Geometric Function Theory, Spectral Theory, and Complex Analysis. Despite its simplicity, even the existence of non-trivial closed invariant subspaces for compact perturbations of self-adjoint operators is still open. The situation is even hopeless if one considers finite rank compact perturbations of normal operators. In this talk we will address these questions and show recent improvements exhibiting broad classes of such perturbations with plenty of non-trivial closed invariant subspaces.

The solution to the $T \oplus T$-recurrence problem

Antoni López-Martínez

Departamento de Matemática Aplicada, Universitat Politècnica de València

alopezmartinez@mat.upv.es

Resumen. Let X be a separable infinite-dimensional F-space. Given a continuous linear operator $T : X \longrightarrow X$ with some dynamical *"property"* (hypercyclicity, chaos, etc.), it is natural to ask whether the direct sum operator

$$T \oplus T : X \oplus X \longrightarrow X \oplus X,$$

acting as $T \oplus T(x_1, x_2) := (Tx_1, Tx_2)$ on the direct sum space $X \oplus X$, presents again that *"property"*. This question will be called the $T \oplus T$-*"property" problem.* In this talk we will discuss the $T \oplus T$-recurrence problem, which was posed as an open question in [4]. In particular, we will motivate the problem by looking at the $T \oplus T$-hypercyclicity problem (see [5, 2, 3]); and then we will expose a complete solution to the $T \oplus T$-recurrence problem in every separable infinite-dimensional Banach space, using a construction from [1] in a crucial way.

References

[1] J.-M. Augé. Linear Operators with Wild Dynamics. *Proc. Amer. Math. Soc.*, **140**(6) (2012), 2103–2116.

[2] F. Bayart and É. Matheron. Hypercyclic operators failing the Hypercyclicity Criterion on classical Banach spaces. *J. Funct. Anal.*, **250**(2) (2007), 426–441.

[3] F. Bayart and É. Matheron. (Non-)Weakly mixing operators and hypercyclicity sets. *Ann. de l'Institut Fourier*, **59**(1) (2009), 1–35.

[4] G. Costakis, A. Manoussos, and I. Parissis. Recurrent linear operators. *Complex Anal. Oper. Theory*, **8**(2014), 1601–1643.

[5] M. De La Rosa and C. Read. A hypercyclic operator whose direct sum $T \oplus T$ is not hypercyclic. *J. Oper. Theory*, **61** (2009), 369–380.

[6] S. Grivaux, A. López-Martínez, and A. Peris. Questions in linear recurrence I: The $T \oplus T$-recurrence problem to lineability. *Anal. Math. Phys.*, **15**(1) (2025), 26 pages.

Agradecimientos. This talk is based on a joint work with Sophie Grivaux and Alfred Peris [6]. This work was partially supported by the project FRONT of the French National Research Agency (grant ANR-17-CE40-0021), by the Labex CEMPI (ANR-11-LABX-0007-01), by the Spanish Ministerio de Ciencia, Innovación y Universidades, grant number FPU2019/04094, by MCIN/AEI/10.13039/501100011033, Projects PID2019-105011GB-I00 and PID2022-139449NB-I00, and by Generalitat Valenciana, Project PROMETEU/2021/070.

Cyclicity of singular inner functions on Besov and coefficient spaces

Alberto Dayan

Department de Matemàtiques, Universitat Autònoma de Barcelona

alberto.dayan@uab.cat

Resumen. The seminal work of Beurling says that no singular inner function can be cyclic for the Hardy space on the unit disc. On the other hand, Korenblum showed that some singular inner functions are cyclic in Bergman-type spaces.
In this talk, we show the existence of singular inner functions that are cyclic in Besov-type spaces. As a corollary, we will see how such singular inner functions are cyclic also for the space of those analytic functions with p-summable Taylor coefficient, for $p > 2$. Our condition relies only on the second modulus of continuity of the underlying singular measure, and hence is far more treatable than the one provided by Anderson, Fernández and Shields in the setting of the small Bloch space. Time permitting, we will also discuss whether such cyclic singular inner function are multipliers of the Besov and the coefficient spaces that we consider.

Agradecimientos. This talk is based on a joint work with Daniel Seco.

Higher order harmonic Schwarzian derivatives

María J. Martín

Departamento de Análisis Matemático & IMAULL, Universidad de La Laguna.

maria.martin@ull.es

Resumen. Let Ω be a domain in the complex plane. A *harmonic mapping* in Ω is a complex valued function $f = u + iv$ whose real and imaginary parts are (real) harmonic in Ω. Analytic functions are a special case where the real and imaginary parts are conjugate harmonic functions, satisfying the Cauchy–Riemann equations. In those cases when the domain Ω is simply connected, the harmonic mapping f can be written as $f = h + \overline{g}$, where both h and g are analytic in Ω.
The landmark paper by James Clunie and Terry Sheil-Small in 1984, points out that many of the classical results for conformal functions have clear analogues for univalent harmonic mappings. However, as Peter Duren mentions in his book on harmonic mappings in the plane, "as soon as analyticity is abandoned, serious obstacles arise".
The purpose of this talk is to show a generalization of some particular objects introduced for analytic functions to those cases when the functions considered are merely harmonic. More concretely, we propose a definition of higher order Schwarzian derivatives for locally univalent harmonic mappings. The definition will be based on the relation between the classical formulas of the pre-Schwarzian and Schwarzian derivatives of locally univalent analytic functions and the derivatives of the generating functions of the methods due to Newton and Halley, respectively, for approximating zeros. We will prove that the higher order harmonic Schwarzian derivatives we obtain coincide, in those cases when the functions considered are holomorphic, with the Aharonov invariants, introduced by Don Aharonov in 1969.

Agradecimientos. This is a joint work with F. Pérez-González and Alexis Quintero-Díaz.

Some questions about weighted Hardy spaces

Luis Rodríguez-Piazza

Departamento de Análisis Matemático & IMUS, Universidad de Sevilla

piazza@us.es

Resumen. Given a sequence $\beta = (\beta_n)_{n\geq 0}$ of positive real numbers (the weights) we define *the weighted Hardy space* $H^2(\beta)$ as the space formed by all functions f analytic at 0 and whose power series at 0, $f(z) = \sum_{n\geq 0} a_n z^n$, satisfies

$$\|f\|^2_{H^2(\beta)} := \sum_{n\geq 0} |a_n|^2 \beta_n < +\infty \,.$$

We will always assume that the sequence of weights (β_n) satisfies the condition

$$\liminf_{n\to\infty} \beta_n^{1/n} \geq 1,$$

and so, every function $f \in H^2(\beta)$ is holomorphic on the whole open unit disc $\mathbb{D}$.
Many classical Hilbert spaces of analytic functions appear as weighted Hardy spaces. For instance, this is the case for Hardy, Bergman and Dirichlet spaces.
If $\varphi\colon \mathbb{D} \to \mathbb{D}$ is holomorphic, the composition operator C_φ of symbol φ is simply

$$C_\varphi\colon f \mapsto f \circ \varphi, \qquad \text{for all } f \in Hol(\mathbb{D}).$$

It is known that, for every symbol φ, C_φ defines a bounded operator on Hardy and Bergman spaces; but this is not true on the Dirichlet space.
In the eighties N. Zorboska raised the following natural question:

> Determine the sequences $\beta = (\beta_n)_{n\geq 0}$ for which every symbol $\varphi\colon \mathbb{D} \to \mathbb{D}$ defines a bounded composition operator $C_\varphi\colon H^2(\beta) \to H^2(\beta)$.

In this talk I will present some results, included in [1] and [2], obtained in collaboration with P. Lefèvre, D. Li and H. Queffélec, dealing with this and other questions about weighted Hardy spaces.

References

[1] P. Lefèvre, D. Li, H. Queffélec, L. Rodríguez-Piazza, *Characterization of weighted Hardy spaces on which all composition operators are bounded.* To appear in Analysis & PDE.

[2] P. Lefèvre, D. Li, H. Queffélec, L. Rodríguez-Piazza, *On some questions about composition operators on weighted Hardy spaces.* To appear in Pure and Applied Functional Analysis.

Surjectivity of the translated weighted composition operators

J. Oliva-Maza

Departamento de Matemáticas, Universidad de Zaragoza

joliva@unizar.es

Resumen.
Weighted composition operators play a crucial role in the study of Banach spaces of holomorphic functions, with one of the main objectives being to describe the operator theoretic properties of uC_φ in terms of the analytic properties of the symbols $u : \mathbb{D} \to \mathbb{C}$ and $\varphi : \mathbb{D} \to \mathbb{D}$. Over the past decade, there have been significant advances in determining the spectrum $\sigma(uC_\varphi)$, usually described through the behavior of u and φ in a neighborhood of the fixed points of φ. In this context, the surjectivity of the translates $\lambda - uC_\varphi$, $\lambda \in \sigma(uC_\varphi)$, was studied in [3], with applications to the theory of universal operators via Caradus' theorem.

In this talk, we present our recent work, in which we obtain the surjectivity of $\lambda - uC_\varphi$, where $\lambda \in uC_\varphi$, φ a hyperbolic Möbius transformation, and u satisfying certain mild hypotheses. Our results constitute an extension of the case $u = 1$, which was solved in [2] for the Hardy space $H^2(\mathbb{D})$. Our approach is completely different from that of [2], which relies on the identification of $H^2(\mathbb{D})$ with $L^2(\mathbb{R})$ and on the results of [4], and is not adaptable to the case $u \neq 1$ or to other spaces. In contrast, our techniques embed certain subspaces of our function space into a space where uC_φ is invertible, and this invertibility allows us to apply the techniques from [1].

References

[1] L. Abadias, J. González-Doña, J. Oliva-Maza (2025). Universality arising from invertible weighted composition operators. *J. Math. Anal. Appl.*, 545, No. 129129.

[2] J. Carmo, W. Noor (2022). Universal Composition Operators. *J. Oper. Theory*, 87(1), No. 137–156.

[3] E. Nordgren, P. Rosenthal and F. S. Wintrobe (1987). Invertible composition operators on H^p. *J. Funct. Anal.*, no. 2, 324–244.

[4] J. R. Partington and E. Pozzi (2011). Universal Shifts and Composition Operators. *Oper. Matrices*, 5 (3), 455–467.

Agradecimientos. Joint work with L. Abadias, J. González-Doña and A. Mahillo.

Boundary behavior of the iterates of a holomorphic self-map of the unit disc

Manuel D. Contreras

Departamento de Matemática Aplicada II and IMUS, Universidad de Sevilla

contreras@us.es

Resumen.
The study of iterated functions is fundamental to complex dynamics. For a holomorphic self-map f of the unit disc $\mathbb{D}$, the Denjoy-Wolff Theorem (1929) establishes a key convergence property: if f is not an elliptic automorphism, its sequence of iterates, $\{f^{\circ n}\}$, converges uniformly on compact subsets to a point $\tau \in \overline{\mathbb{D}}$, called the Denjoy-Wolff point of f.
A related question concerns the behavior of these iterates at the boundary. By Fatou's Theorem, all functions $f^{\circ n}$ have non-tangential limits at almost every point on the boundary of the unit disc. We denote these non-tangential limits as $(f^{\circ n})^*$. The behavior of the sequence $\{(f^{\circ n})^*\}$ depends significantly on whether f is an inner function or not. When f is an inner function, the behaviour of $\{(f^{\circ n})^*\}$ is well-established and can be found in texts by Anderson or Doering and Mañe.
However, when f is not an inner function, the problem was not solved. Previous partial results have been contributed by Bourdon, Matache, and Shapiro; by Poggi-Corradini; and by Contreras, Díaz-Madrigal, and Pommerenke. In a recent work with Betsakos and Díaz-Madrigal, we achieved the final solution: if f is not an inner function, then the sequence $\{(f^{\circ n})^*\}$ converges to τ for almost every point on $\partial\mathbb{D}$.

The rate of convergence of orbits in complex dynamics

Francisco J. Cruz-Zamorano

Departamento de Matemática Aplicada II, Universidad de Sevilla

fcruz4@us.es

Resumen. Discrete Complex Dynamics in the unit disk $\mathbb{D}$ studies the asymptotic behaviour of the iterates $f^n = f \circ \overset{(n)}{\cdots} \circ f$ of a given holomorphic self-map $f\colon \mathbb{D} \to \mathbb{D}$. A seminal result due to Denjoy and Wolff (1929) established that, if f has no fixed points in $\mathbb{D}$, then there exists a (unique) point $\tau \in \partial\mathbb{D}$ (known as the Denjoy-Wolff point of f) such that $f^n \to \tau$ locally uniformly in $\mathbb{D}$. Consequently, τ serves as a global attractor for the dynamics of f.
Recent research has focused on quantifying the rate of convergence of orbits $(f^n(z))$ towards the Denjoy-Wolff point τ, that is, the asymptotic behavior of $|f^n(z) - \tau|$ as $n \to \infty$. This talk presents a characterization of those self-maps f exhibiting an extremal rate of convergence (i.e., maps for which $f^n(z)$ approaches τ at the slowest possible rate). For instance, it turns out that this extremal behavior is determined by specific integrability conditions on the Herglotz representation associated with f.

Agradecimientos. This is a joint work with Konstantinos Zarvalis (Aristotle University of Thessaloniki, Greece).

Cèsaro-type operators on mixed norm spaces

Oscar Blasco

Departmento de Análisis Matemático, Universidad de Valencia

oscar.blasco@uv.es

Resumen.
Given a positive Borel measure μ defined in $[0,1)$ and $\beta > 0$, we denote by

$$\mathcal{C}_{\mu,\beta}(f)(z) = \sum_{n=0}^{\infty} \mu_n (\sum_{k=0}^{n} \frac{\Gamma(n-k+\beta)}{(n-k)!\Gamma(\beta)} a_k) z^n$$

where $\mu_n = \int_0^1 t^n d\mu(t)$ and $f(z) = \sum_{n=0}^{\infty} a_n z^n$. It is elementary to see that

$$\mathcal{C}_{\mu,\beta}(f)(z) = \int_0^1 \frac{f(tz)}{(1-tz)^{\beta}} d\mu(t).$$

We shall try to describe the boundedness of $\mathcal{C}_{\mu,\beta}$ acting between different mixed norm space $H(p,q,\alpha)$ consisting in functions $f \in \mathcal{H}(\mathbb{D})$ such that

$$\|f\|_{p,q,\alpha} = (\int_0^1 (1-r)^{q\alpha-1} M_p^q(f,r) dr)^{1/q} < \infty.$$

Complete characterization is achieved in several cases in terms of properties of the function $F_\mu(z) = \sum_{n=0}^{\infty} \mu_n z^n$. This extends previously known results on weighted Bergman spaces.

References

[1] P. Ganalopoulos, D. Girela, N. Merchan, *Cèsaro-like operators acting on spaces of analytic functions, Anal. Math. Phys.* **12** (51), 2022.

[2] P. Galanopoulos, A.G. Siskakis, R. Zhao, *Weighted Cesàro type operators between weighted Bergman spaces Bull. Sci. Math.* 202 (2025), Paper No. 103622, 17 pp.

Agradecimientos. Joint work with Alejandro Mas.

Rhaly operators acting on spaces of analytic functions

Daniel Girela

Departamento de Análisis Matemático, Universidad de Málaga

girela@uma.es

Resumen. Let $\mathcal{S}$ be the space of all complex sequences. If $(\eta) = \{\eta_n\}_{n=0}^{\infty} \in \mathcal{S}$, the Rhaly operator $R_{(\eta)} = R_{\{\eta_n\}}$ is defined in $\mathcal{S}$ as follows: If $\{a_n\}_{n=0}^{\infty} \in \mathcal{S}$, then

$$R_{\{\eta_n\}}(\{a_n\}_{n=0}^{\infty}) = \left\{\eta_n \sum_{k=0}^{n} a_k\right\}_{n=0}^{\infty}.$$

Rhaly operators are a natural generalization of the Cesàro operator.
Identifying a function f, analytic in the unit disc $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$, with the sequence of its Taylor coefficients, $R_{(\eta)}$ can be seen as an operator acting on spaces of analytic functions in $\mathbb{D}$: If f is holomorphic in $\mathbb{D}$, $f(z) = \sum_{n=0}^{\infty} a_n z^n$ $(z \in \mathbb{D})$, then $R_{(\eta)}(f)$ is formally defined by

$$R_{(\eta)}(f)(z) = \sum_{n=0}^{\infty} \eta_n \left(\sum_{k=0}^{n} a_k\right) z^n, \quad z \in \mathbb{D},$$

whenever the right hand side makes sense and defines an analytic function in $\mathbb{D}$.
In this talk I will speak about a number of results obtained recently in collaboration with Petros Galanopoulos, regarding the operators $R_{(\eta)}$ acting on distinct spaces of analytic functions in $\mathbb{D}$.

Words of analytic paraproducts on Bergman spaces induced by smooth rapidly decreasing weights

Carme Cascante Canut

Departament de Matemàtiques i Informàtica, Universitat de Barcelona

cascante@ub.edu

Resumen.
For a fixed analytic function g on the unit disc, we consider the analytic paraproducts induced by g, which are formally defined by $T_g f(z) = \int_0^z f(\zeta) g'(\zeta) d\zeta$, $S_g f(z) = \int_0^z f'(\zeta) g(\zeta) d\zeta$, and $M_g f(z) = g(z) f(z)$. An N-letter g-word is an operator of the form $L = L_1 \cdots L_N$, where each L_j is either M_g, S_g or T_g. It has been recently proved in [1, 2] that understanding the boundedness of a g-word on classical Hardy and Bergman spaces is a challenging problem due to the potential cancellations involved. Our main result provides a complete quantitative characterization of the boundedness of an arbitrary g-word on a weighted Bergman space $A^p_{\omega^{p/2}}$, where $\omega = e^{-2\varphi}$ is a smooth rapidly decreasing weight. In particular, it states that any N-letter g-word such that $\#\{j : L_j = T_g\} = n \geq 1$ is bounded on $A^p_{\omega^{p/2}}$ if and only if g satisfies the "fractional" Bloch-type condition

$$\|g\|^s_{\mathscr{B}^s_\varphi} := \sup_{z \in \mathbb{D}} \frac{s|g(z)|^{s-1}|g'(z)|}{1 + \varphi'(|z|)} < \infty,$$

where $s = \frac{N}{n}$, and $\|L\|_{A^p_{\omega^{p/2}}} \simeq \|g\|^N_{\mathscr{B}^s_\varphi}$.
The class of smooth rapidly decreasing weights contains the radial weights

$$\omega_n(z) = e^{-2\exp_n(g_{\alpha,c}(|z|))}, \quad \text{where} \quad g_{\alpha,c}(r) = \frac{c}{(1-r^2)^\alpha}, \quad \text{for } c, \alpha > 0,$$

$\exp_0(x) = x$ and $\exp_n(x) = e^{\exp_{n-1}(x)}$, for $n \in \mathbb{N}$. Therefore it contains weights which decrease arbitrarily rapidly to zero as $|z| \to 1^-$.

References

[1] A. Aleman, C. Cascante, J. Fàbrega, D. Pascuas, and J. A. Peláez, *Composition of analytic paraproducts*, J. Math. Pures Appl. 158 (2022), 293–319. doi:10.1016/j.matpur.2021.11.007.

[2] A. Aleman, C. Cascante, J. Fàbrega, D. Pascuas, and J. A. Peláez, *Words of analytic paraproducts on Hardy and weihted Bergman spaces*, J. Math. Pures Appl. 188 (2024), 179–214. doi:10.1016/j.matpur.2024.05.002.

[3] C. Cascante, J. Fàbrega, D. Pascuas, and J. A. Peláez, *On the radicality property for spaces of symbols of bounded Volterra operators*, J. Funct. Anal. 287 (2024), no.12, Paper No. 110658, 29 pages. doi:10.1016/j.jfa.2024.110658.

[4] C. Cascante, J. Fàbrega, D. Pascuas, and J. A. Peláez, *Words of analytic paraproducts on Bergman spaces induced by smooth rapidly decreasing weights*, (preprint). arxiv.org/abs/2504.09105.

Agradecimientos. Joint work with J. Fàbrega, D. Pascuas and J.A. Peláez.

Fractional Volterra-type operator

Álvaro Miguel Moreno López

Departamento Análisis Matemático, Estadística e I.O. y Matemática Aplicada, Universidad de Málaga

alvarommorenolopez@uma.es

Resumen. Let $\mathcal{H}(\mathbb{D})$ denote the space of analytic functions on the unit disk $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$. Essential properties such as boundedness, compactness and the membership to Schatten classes of the classical Volterra-type operator

$$V_g(f)(z) = \int_0^z f(\zeta)g'(\zeta)\, d\zeta, \quad z \in \mathbb{D},\ f \in \mathcal{H}(\mathbb{D}),$$

were studied on classical Hardy and Bergman spaces in the seminal papers [1, 2, 3]. In this talk, we consider a generalization of this operator where the derivative and primitive are replaced by a fractional derivative and primitive. Being precise, given a radial weight μ on $\mathbb{D}$, and a function $f(z) = \sum_{n=0}^{\infty} \widehat{f}(n) z^n \in \mathcal{H}(\mathbb{D})$, we define the fractional derivative and integral operators:

$$D^\mu(f)(z) = \sum_{n=0}^{\infty} \frac{\widehat{f}(n)}{\mu_{2n+1}} z^n, \quad I^\mu(f)(z) = \sum_{n=0}^{\infty} \mu_{2n+1} \widehat{f}(n) z^n, \quad z \in \mathbb{D},$$

where $\mu_x = \int_0^1 \mu(r) r^x\, dr$. Using these operators, we consider the fractional Volterra-type operator

$$V_{\mu,g}(f)(z) = I^\mu(f \cdot D^\mu(g))(z), \quad f \in \mathcal{H}(\mathbb{D}),\ z \in \mathbb{D}.$$

Throughout the talk it will be presented some results on essential properties of the action the $V_{\mu,g}$ on classical Hardy and Bergman spaces

References

[1] A. Aleman and J. A. Cima, *An integral operator on H^p and Hardy's inequality*, J. Anal. Math. **85** (2001), 157–176; MR1869606

[2] A. Aleman and A. Siskakis, *An integral operator on H^p*, Complex Variables Theory Appl. **28** (1995), no. 2, 149–158.

[3] A. Aleman and A. G. Siskakis, *Integration operators on Bergman spaces*, Indiana Univ. Math. J. **46** (1997), no. 2, 337–356; MR1481594

Agradecimientos. These results are part of a joint work together with C. Bellavita, G. Nikolaidis and J. A. Peláez.

On the range of Toeplitz operators

Bozhidar E. Mihaylov

Departamento de Matemáticas, Universidad Autónoma de Madrid

bozhidar.emilov@estudiante.uam.es

Resumen.
A Toeplitz operator acting on the Hardy space H^2 is defined as the composition of a multiplication operator with symbol $\phi \in L^\infty$ and the Riesz orthogonal projection. One property of these operators is that, given a nonzero and non-injective Toeplitz operator T_ϕ, its range contains the polynomials, which refines Coburn's Lemma.
In this talk (based on ongoing work), we present a characterization, in terms of the symbol ϕ, of Toeplitz operators whose range contains the polynomials, and we use it to give a characterization of those operators that have a nontrivial kernel. Finally, we discuss some applications of these results to the study of the spectrum of general Toeplitz operators.

References

[1] A. Brown, P. Halmos: Algebraic Properties of Toeplitz Operators *J. Reine Angew. Math.* ***213*** *(1963), 89-102.*

[2] L. Coburn: Weyl's Theorem for Nonnormal Operators. *Michigan Math. J.* ***13*** *(1966), 285-288.*

[3] R. Douglas: Banach Algebra Techniques in Operator Theory, *Springer, New York* 1998.

[4] A. Hartmann, M. Mitkovski: Kernels of Toeplitz operators, Recent progress on operator theory and approximation in spaces of analytic functions, 147–177, *Contemp. Math.* **679**, *Amer. Math. Soc., Providence, RI* 2016.

[5] P. Koosis: Introduction to H^p spaces. *Cambridge University Press, Cambridge* 1988.

[6] R. Martínez-Avendaño, P. Rosenthal: An Introduction to Operators on the Hardy-Hilbert Space. *Springer, New York* 2007.

[7] D. Sarason: Sub-Hardy Hilbert Spaces in the Unit Disk. *University of Arkansas Lecture Notes in the Mathematical Sciences* ***10***, *John Wiley & Sons Inc., New York,* 1994.

[8] D. Vukotić: A Note On The Range of Toeplitz Operators. *Integral Equations Operator Theory* ***50*** *(2004), 565-567.*

Agradecimientos. Joint work with D. Vukotić.