EDPs: problemas no lineales y aplicaciones

Equipo organizador

- José C. Sabina de Lis (Universidad de La Laguna)
- Sergio Segura de León (Universitat de València)

Descripción

En la sesión se aborda una panorámica actual de la investigación matemática española en el campo de la ecuaciones diferenciales. Naturalmente el énfasis se centra en ecuaciones en derivadas parciales elípticas y parabólicas. La temática propuesta comprende una gran variedad de temas, como se puede comprobar por los títulos y resúmenes de las comunicaciones.

Conviene señalar que la propuesta reúne un plantel de investigadores tanto noveles como de reconocida experiencia en el campo de las ecuaciones no lineales. Se abarca asimismo una muestra representativa de los diversos grupos y centros de investigación del país.

Esperamos que la sesión genere un foro de discusión que dé impulso a nueva investigación en la disciplina.

Palabras clave: ecuaciones no locales; dinámica de poblaciones; fenómenos de concentración; difusión lineal y no lineal; exponentes críticos y desarrollo de singularidades.

Programa

Lunes, 19 de enero

15:30 – 16:00 Fernando Quirós (Universidad Autónoma de Madrid)
Widder-type results for nonlocal heat equations

16:00 – 16:30 Marta Latorre (Universidad Rey Juan Carlos)
Study of a parabolic problem involving the 1–Laplacian operator and integrable data

16:30 – 17:00 Martina Magliocca (Universidad de Sevilla)
Global existence for certain fourth order evolution equations

17:00 – 17:30 Juan Campos (Universidad de Granada)
The shape of an epidemic wave

Martes, 20 de enero

11:00 – 11:30 Mabel Cuesta (Université du Littoral Côte d'Opale)
A p-Laplacian problem with slightly subcritical regularly varying nonlinearity

11:30 – 12:00 Alexis Molino Salas (Universidad de Almería)
Existence of nontrivial solutions for an autonomous semilinear elliptic equation

12:00 – 12:30 Raul Ferreira (Universidad Complutense de Madrid)
Blow-up for a fully fractional heat equation

15:30 – 16:00 Rosa Pardo (Universidad Complutense de Madrid)
Elliptic Systems with Superlinear Terms on the Critical Hyperbola

16:00 – 16:30 Pedro Martínez-Aparicio (Universidad de Almería)
Unexpected results for a singular elliptic problem

16:30 – 17:00 Salvador López Martínez (Universidad Autónoma de Madrid)
Dark-bright solitons to one-dimensional Gross-Pitaevskii systems

17:00 – 17:30 Antonio Suárez (Universidad de Sevilla)
Semilinear interface elliptic equation arising in population dynamics

The shape of an epidemic wave

Juan Campos

Departamento de Matemática Aplicada, Universidad de Granada

campos@ugr.es

Resumen. The recent Covid-19 epidemic has made us see what an epidemic looks like. The most basic way to simulate these waves is to use the model described in [2]. This compartimental model was presented based on three groups of individuals, *susceptible, infected* and *recovered* and lead a differential system with three equations

$$\begin{aligned} S' &= -\beta SI, \\ I' &= \beta SI - \delta I, \\ R' &= \delta I. \end{aligned}$$

where $\beta > 0$ and $\delta > 0$ are the standard rates of infection and recovery, respectively. At first glance, only the first two equations are necessary. That is, they can be reduced to the study of susceptibles and infected. However, has anyone seen a profile of susceptibles within the numerous graphs in the news that the recent pandemic provided? The concept of a susceptible individual is not clear. Individuals who are either well confined or, for some genetic or biological reason, give rise to individuals who cannot suffer from the disease.
A method is proposed to understand biological parameters that only uses the profiles of infected individuals or at most recovered individuals. New parameters that are: epidemic severity, wave asymmetry and fraction of endemicity can be used to recoverind the infected profile. These results follows from [1].

Referencias

[1] J. Campos, M.C.A. Leite. Novel Approach to Extract Epidemiological Information from Waves in Epidemic's Profiles. to appear in Infectious Disease Modelling.

[2] W. Kermack, A. McKendrick. (1991). Contributions to the mathematical theory of epidemics – I, *Bulletin of Mathematical Biology* 53 (1–2) 33–55.

Agradecimientos. Proyecto parcialmente financiado por el Proyecto PID2022–137228OB–I00 financiado por el Ministerio de Ciencia, MICIU/AEI/10.13–039/501100011033 y "ERDF/EU A way of making Europe", pot el Proyecto C–EXP–265–UGR23 financiado por Consejería de Universidad, Investigación e Innovación & ERDF/EU Andalusia Program, & by Modeling Nature Research Unit, project QUAL21–011.

A p-Laplacian problem with slightly subcritical regularly varying nonlinearity

Mabel Cuesta, Rosa Pardo

Université du Littoral Côte d'Opale, Calais (France)

maria-mabel.cuesta-leon@univ-littoral.fr

Resumen.
We study the following quasilinear elliptic problem involving the p-laplacian operator and a slightly subcritical nonlinearity with a sign-changing weight $m \in C^1(\Omega)$:

$$\begin{cases} -\Delta_p u = \lambda u^{p-1} + m(x) f(u), & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a smooth bounded domain of $\mathbb{R}^N$, $1 < p < N$ and Δ_p is the $p-$Laplacian operator. We assume that the slightly subcritical nonlinearity f is a regularly varying function at zero and at infinity, which are not necessarily asymptotic to a power at infinity. We state sufficient conditions which guarantee a Palais-Smale condition. We also provide a bifurcation theorem for these nonlinearities, which allow us to state the existence of a bifurcated branch of positive solutions, containing a turning point, and multiplicity of solutions.

A model of nonlinearity f that we have in mind is $f(s) = \frac{s^{p^*-1}}{\ln(e+s)^\alpha}, \quad \alpha > 0,\ p^* = \frac{Np}{N-p}$

Agradecimientos. Trabajo en colaboración con Rosa Pardo (Universidad Complutense de Madrid).

Blow-up for a fully fractional heat equation

Raúl Ferreira, Arturo de Pablo

Departamento de Análisis Matemático y Matemática Aplicada, Universidad Complutense de Madrid

raul ferreira@mat.ucm.es

Resumen. We study the existence and behaviour of blowing-up solutions to the fully fractional heat equation

$$\mathcal{M}u = u_p, \quad x \in \mathbb{R}^N, 0 < t < T$$

with $p > 0$, where $\mathcal{M}$ is a nonlocal operator given by a space-time kernel

$$M(x,t) = c_{N,\sigma} t^{-\frac{N}{2}-1-\sigma} e^{-\frac{|x|^2}{4}t} \mathbb{1}_{\{t>0\}}, \quad 0 < \sigma < 1.$$

This operator coincides with the fractional power of the heat operator, $\mathcal{M} = (\partial_t - \Delta)^\sigma$ defined through semigroup theory.

Agradecimientos. Trabajo en colaboración con Arturo de Pablo.

Study of a parabolic problem involving the 1–Laplacian operator and integrable data

Marta Latorre, Sergio Segura de León

Departamento de Matemática Aplicada, Ciencia e Ingeniería de los Materiales y Tecnología Electrónica, Universidad Rey Juan Carlos

marta.latorre@urjc.es

Resumen. In this talk, we consider a parabolic problem involving the 1–Laplacian operator in a bounded domain $\Omega \subset \mathbb{R}^N$ with Lipschitz boundary, subject to Dirichlet boundary conditions and L^1–data. Specifically, we consider the problem

$$\begin{cases} u'(t,x) - \Delta_1 u(t,x) = f(t,x) & \text{in } (0,T)\times\Omega, \\ u(t,x) = 0 & \text{on } (0,T)\times\partial\Omega, \\ u(0,x) = u_0(x) & \text{en } \Omega, \end{cases}$$

where $T > 0$, and both the initial data u_0 and the source term f are merely integrable functions. The low regularity of the data prevents the existence of weak solutions.
To address this difficulty, we use the notion of entropy solutions and prove the existence and uniqueness of this type of solutions through an approximation procedure with regular data and the use of truncations. In addition, we establish a comparison principle and analyze the additional regularity that solutions present when data belong to spaces L^r with $1 < r < 2$.

Agradecimientos. Trabajo en colaboración con Sergio Segura de León. Proyecto PID2022-136589NB-I00 parcialmente financiado por MCIN/ AEI/10.13039/501100011033/ y por FEDER Una manera de hacer Europa.

Dark-bright solitons to one-dimensional Gross-Pitaevskii systems

Salvador López Martínez

Departamento de Matemáticas, Universidad Autónoma de Madrid

salvador.lopez@uam.es

Resumen. Solitons are localized structures that propagate at constant speed without changing shape. A visual analogy would be a traveling water wave in a narrow channel. These structures typically arise in physical systems governed by coupled Gross-Pitaevskii equations, such as mixtures of Bose-Einstein condensates (BECs) and multimode nonlinear optics. Solitons are classified as dark or bright, depending on whether the density is a positive constant or zero at infinity, and emerge under repulsive (defocusing) or attractive (focusing) interactions, respectively. Experimental and numerical studies confirm the existence of stable dark-bright solitons, even when self-defocusing interactions are present in the bright component. In this talk, we will present recent analytical results that rigorously validate this phenomenon. Specifically, we will show that symmetric and radially monotone (in modulus) dark-bright solitons can be derived as energy minimizers subject to a momentum-mass constraint.

Global existence for certain fourth order evolution equations

Martina Magliocca, R. Granero-Belinchón

Departamento de Análisis Matemático, Universidad de Sevilla

mmagliocca@us.es

Resumen. In this talk, we will see three global in time results for two fourth order nonlinear parabolic equations. The first of such equations involves the Hessian and appears in epitaxial growth. For such an equation, we give conditions ensuring the global existence of the solution. For certain regime of the parameters, our size condition involves the norm in a critical space with respect to the scaling of the equation and improves previous existing results in the literature for this equation. The second of the equations under study is a thin film equation with a porous medium nonlinearity. For this equation, we establish conditions leading to the global existence of the solution.

Agradecimientos. Trabajo en colaboración con R. Granero-Belinchón. Parcialmente financiado por los proyectos PID2022-141187NB-I00 y PID2022-140494NA-I00 financiados por MCIN /AEI /10.13039/501100011033 / FEDER, UE.

Unexpected results for a singular elliptic problem

Pedro J. Martínez–Aparicio, Daniela Giachetti, François Murat, Francesco Petitta

Departamento de Matemáticas, Universidad de Almería

pma624@ual.es

Resumen.
In this lecture, I will present recent results [1, 2, 3] obtained in collaboration with Daniela Giachetti (Universitá di Roma, Sapienza), Francois Murat (Sorbonne Université, París), and Francesco Petitta (Universitá di Roma, Sapienza), for the one-dimensional singular boundary value problem

$$-\frac{d}{dx}\Big(a(x)\frac{du(x)}{dx}\Big) = -\frac{d\phi(u(x))}{dx} - \frac{dg(x)}{dx} \text{ in } (0,L), \quad u(0) = u(L) = 0,$$

where the model for the singular function ϕ is $\phi(s) = \dfrac{1}{|s|^\gamma}$ with $\gamma > 0$.

This singular problem presents a number of unexpected phenomena: nonexistence of solutions under certain assumptions, existence of an infinite number of solutions under other assumptions, and non-continuity of the solution with respect to the data.

Referencias

[1] D. Giachetti, P. J. Martínez-Aparicio, F. Murat and F. Petitta (2025) Unexpected phenomena in a one-dimensional elliptic equation with a singular first order divergence term. Preprint.

[2] D. Giachetti, P. J. Martínez-Aparicio, F. Murat and F. Petitta. Remarks and variants on a one-dimensional elliptic equation with a singular first order divergence term, in preparation.

[3] D. Giachetti, P. J. Martínez-Aparicio, F. Murat and F. Petitta A one-dimensional elliptic equation with a first order divergence term singular in $m \neq 0$, in preparation.

Agradecimientos. Trabajo en colaboración con Daniela Giachetti, François Murat y Francesco Petitta. Parcialmente financiado por el proyecto PGC2018-096422-B-I00 y por el Grupo de la Junta de Andalucía FQM-116..

Existence of nontrivial solutions for an autonomous semilinear elliptic equation

Alexis Molino, Salvador Villegas

Departamento de Matemáticas, Universidad de Almería

amolino@ual.es

Resumen. This talk is concerned with the existence of nontrivial solutions to the problem

$$\begin{cases} -\Delta u &= \lambda u + f(u) & \text{in } \Omega, \\ u &= 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a smooth bounded domain, $N \geq 1$, $\lambda \in \mathbb{R}$ and $f : \mathbb{R} \to \mathbb{R}$ is any locally Lipschitz function with nonpositive primitive.
A complete description is obtained for $N = 1$ and partial results for $N \geq 2$.

Referencias

[1] A. Molino, S. Villegas (2025) Solutions for autonomous semilinear elliptic equations (2025). Submitted. https://arxiv.org/pdf/2504.18877

[2] S. López-Martínez, A. Molino (2022) Nonexistence result of nontrivial solutions to the equation $-\Delta u = f(u)$, *Complex Var. Elliptic Equ.* 67, no. 1, 239–245.

Agradecimientos. Trabajo en colaboración con Salvador Villegas. Proyecto parcialmente financiado por (MCIU) Ministerio de Ciencia, Innovación y Universidades, Agencia Estatal de Investigación (AEI) y Fondo Europeo de Desarrollo Regional a través del proyecto PID2021- 122122NB-I00 (FEDER) y por Junta de Andalucía FQM-116.

Elliptic Systems with Superlinear Terms on the Critical Hyperbola

Rosa Pardo, Mabel Cuesta, Angela Pistoia

Departamento de Análisis Matemático y Matemática Aplicada, Universidad Complutense de Madrid

rpardo@ucm.es

Resumen. We focus on semilinear elliptic systems involving a nonlinearity of slightly subcritical nature understood in a generalized sense. For this problem, standard compact embeddings cannot be used to guarantee the existence of solutions as in the case of power-type nonlinearities. Instead, we use the dual method on Orlicz spaces. Roughly speaking, this method consists in taking the inverse of the Laplace operator, and defining the inverse of the nonlinearities. We state sufficient conditions guaranteeing the Palais-Smale condition, showing that our problem possesses a mountain pass type solution.

Referencias

[1] M. Cuesta, R. Pardo, A. Pistoia. (2024). Positive solutions of elliptic systems with superlinear terms on the critical hyperbola. *Milan J. Math.*, 92(2), 439–472.

Agradecimientos. Trabajo en colaboración con Mabel Cuesta y Angela Pistoia. Proyecto parcialmente financiado por PID2022-137074NB-I00, MICINN, España, y por UCM, España, Grupo 920894.

Widder-type results for nonlocal heat equations

Fernando Quirós, Irene González, Fernando Soria, Zoran Vondraček

Departamento de Matemáticas, Universidad Autónoma de Madrid
Instituto de Ciencias Matemáticas ICMAT (CSIC-UAM-UCM-UC3M)

fernando.quiros@uam.es

Resumen. We develop a Widder-type theory for nonlocal heat equations involving quite general Lévy operators. Thus, we consider nonnegative solutions and look for conditions on the operator that ensure: (i) uniqueness of nonnegative classical and very weak solutions with a given initial trace; (ii) the existence of an initial trace, belonging to certain admissibility class; and (iii) the existence of a solution, given by a representation formula, for any admissible initial trace. Such results are obtained first for purely nonlocal Lévy operators defined through positive symmetric Lévy kernels comparable to radial functions with mixed polynomial growth, and then extended to more general operators, including anisotropic ones and operators that have both a local and a nonlocal part.

Agradecimientos. Trabajo en colaboración con Irene Gonzálvez (UAM/ICMAT), Fernando Soria (UAM) y Zoran Vondraček (Dr. Franjo Tuđman Defense and Security U. /U. Zagreb). Parcialmente financiado por MICIU/AEI/10.13039/501100011033 por medio de los proyectos PID2023-146931NB-I00, RED2022-134784-T y CEX2023-001347-S.

Semilinear interface elliptic equation arising in population dynamics

Antonio Suarez, C. Morales–Rodrigo, M. Molina–Becerra, B. B. V. Maia, P. Álvarez–Caudevilla, Cristina Brändle

Departamento de Ecuaciones Diferenciales y Análisis Numérico, Universidad de Sevilla

suarez@us.es

Resumen. In this talk we consider an interface logistic problem where two populations live in two different regions, separated by a membrane or interface where it happens an interchange of flux. Thus, the two populations only interact or are coupled through such a membrane where we impose the so-called Kedem-Katchalsky boundary conditions. For this particular scenario we analyze the existence and uniqueness of positive solutions depending on the parameters involved in the system.

Agradecimientos. Trabajo en colaboración con C. Morales-Rodrigo (US), M. Molina-Becerra (US), B. B. V. Maia (UFRA), P. Álvarez-Caudevilla (UCIII) y Cristina Brändle (UCIII). Proyecto parcialmente financiado por el Proyecto PID2023-149509NB-I00 (Ministerio de Ciencia, Innovación y Universidades)