Estructuras superiores en Álgebra, Geometría y Topología

Equipo organizador

- Víctor Carmona (Max-Planck Institute for Mathematics in the Sciences)
- Fernando Muro (Universidad de Sevilla)

Descripción

Esta sesión está dedicada al estudio de estructuras superiores que surgen en el álgebra, la geometría y la topología, y que desempeñan un papel central en el desarrollo de teorías matemáticas contemporáneas. Por citar algunos ejemplos, estas son fundamentales para la teoría de categorías superiores, la teoría de homotopía estable e inestable y en geometría derivada. Se pretende establecer un marco de discusión común en el que investigadores de diversas areas puedan encontrar un lenguaje compartido mediante la variedad de técnicas empleadas, fomentando así nuevas sinergias y posibles colaboraciones. En particular, se prestará especial atención a las opéradas, a sus variaciones y a sus aplicaciones.

Palabras clave: higher structures; (pr)operads; higher categories.

Programa

Lunes, 19 de enero

15:30 – 16:00	Ana Ros Camacho (Cardiff University) *Extensions of Tambara-Yamagami categories and their NIM-representations*
16:00 – 16:30	Imma Gálvez Carrillo (Universidad de Málaga) *Symmetric functions, objectively*
16:30 – 17:00	Javier J. Gutiérrez (Universitat de Barcelona) *A model for ∞-operads based on dendroidal Segal spaces*
17:00 – 17:30	Andrew Tonks (Universidad de Málaga) *Some remarks on Markl's twisted structures*

Martes, 20 de enero

11:00 – 11:30	Mario Fuentes (Institut de Mathématiques de Toulouse) *Associativity or commutativity, but not both*
11:30 – 12:00	Cristina Costoya (Universidad de Santiago de Compostela) *Realizing finite subsets of $\mathbb{Q}$ as mapping degree sets*
12:00 – 12:30	Aniceto Murillo (Universidad de Málaga) *Teoría de homotopía estable racional*

15:30 – 16:00	Alexandre Quesney (Universidad Politécnica de Madrid) *Estructuras en geometría no-conmutativa*
16:00 – 16:30	Víctor Roca i Lucio (Université Paris Cité) *The algebraic structures of social organizations: the operad of cooperative games*
16:30 – 17:00	José M. Moreno Fernández (Universidad de Málaga) *Unos productos de Massey sensibles a la estructura conmutativa en términos de determinantes*

Extensions of Tambara-Yamagami categories and their NIM-representations

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Resumen. Tambara-Yamagami categories are one of the easiest instances of near-group fusion categories, and they have been studied in depth. In particular, they are connected to different topics in mathematical physics (like e.g. conformal nets), making them even more interesting. Several ways of extending Tambara-Yamagami categories have been introduced by diverse authors in recent years [4, 1, 2]. In this talk we will present these, the respective fusion rings, how to compute their non-negative integer matrix representations (or NIM-reps for short), and how to use these to classify algebra objects in these categories based on [3]. This is joint work in progress with A. Czenky, E. McGovern, M. Molander and M. Müller as part of the 'Women in Mathematical Physics 3' initiative.

Referencias

[1] C. Galindo, S. Lentner, S. Möller (2024). Modular $\mathbb{Z}_2$-Crossed Tambara-Yamagami-like Categories for Even Groups. *arXiv: 2411.12251.*

[2] C. Galindo, S. Lentner, S. Möller (2024). Computing G-Crossed Extensions and Orbifolds of Vertex Operator Algebras. *arXiv: 2409.16357.*

[3] S. Hannah, A. Ros Camacho, Appendix with D. Young (2024). Detecting algebra objects from NIM-reps in pointed, near-group and quantum group-like fusion categories. *Nuclear Physics B*, Volume 1002, 116525.

[4] D. Jordan, E. Larson (2009). On the classification of certain fusion categories. *J. Noncommut. Geom.*, 3, 481–499.

Symmetric functions, objectively

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Resumen. In this talk I will report on progress with J. Kock and A. Tonks on examples of decomposition spaces [2, 3] (also known as higher Segal spaces [1]) whose homotopy cardinalities recover the combinatorial Hopf algebras of symmetric functions. This provides both a fresh perspective on symmetric functions as well as highlighting some fundamental ideas in the theory of decomposition spaces.

We first introduce decomposition spaces $\Lambda^M, \Lambda^P, \Lambda^H$ etc, the objective counterparts of the ring of symmetric functions in the classical $m, p, h, \dots$ bases. These cases arise from monoidal restriction species and so have compatible algebra and coalgebra structures. They are not isomorphic or even homotopy equivalent (so we can say the objective theory is finer than the algebraic setting) and our next step is to provide linear functors between them — the objective counterparts of change-of-basis matrices — and here we find important examples 'in nature' of bialgebra decomposition spaces that do not arise from monoidal restriction species.

Time permitting we will discuss objective antipodes, Möbius invertibility of the change-of-basis functors, or relations with other classical combinatorial structures, such as the Malvenuto–Reutehauer or Loday–Ronco Hopf algebras, arising from permutations or trees.

Referencias

[1] T. Dyckerhoff, P. M. Kapranov (2019). *Higher Segal Spaces.* Springer.

[2] I. Gálvez-Carrillo, J. Kock, A. Tonks (2018) Decomposition spaces, incidence algebras and Möbius inversion I: Basic theory. *Adv. Math.* 331, 952–1015.

[3] I. Gálvez-Carrillo, J. Kock, A. Tonks. Decomposition spaces in combinatorics. *To appear.*

A model for ∞-operads based on dendroidal Segal spaces

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Resumen. In this talk, we present a model structure for ∞-operads on the category of dendroidal spaces, whose fibrant objects are precisely the dendroidal Segal spaces, that is, dendroidal spaces satisfying the Segal condition without imposing completeness. We show that this model structure is Quillen equivalent to the model of ∞-operads given by complete dendroidal Segal spaces. Moreover, slicing over the dendroidal space of the trivial tree recovers the analogous model structure for ∞-categories in simplicial spaces recently established by Nuiten and Moser. This is a joint work with João Candeias.

Some remarks on Markl's twisted structures

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Resumen. This talk will be an introduction to our recently published paper 'On differential Hopf algebras and B_∞ algebras' [1] joint with Imma Gálvez-Carrillo and María Ronco, and also some possible (more speculative) consequences. The title refers to Martin Markl's 2015 paper [3] on the origin of higher braces and higher order derivations, where he observed the L_∞ Koszul hierarchy and the Börjeson A_∞ hierarchy are examples of 'trivial' structures twisted by a canonical automorphism. Thus, given a trivial A_∞ algebra $(A, d, 0, ...)$, one may conjugate the corresponding differential ∂ on the bar construction by any coalgebra automorphism f of BA and obtain a new A_∞ algebra structure. Our contribution was to observe, firstly, that analogous formalism gives immediately the multibrace algebra construction of Loday–Ronco [2] which together with the rigidity theorem for (conilpotent) unital infinitesimal biaglebras gives a Milnor-Moore style equivalence of categories, and, secondly, that the Börjeson and Loday–Ronco results combine to give a new Milnor-Moore theorem for B_∞ and differential 2-associative bialgebras.

Referencias

[1] I. Gálvez-Carrillo, M. Ronco, A. Tonks (2025). On differential Hopf algebras and B_∞-algebras. *Mediterr. J. Math.* 22, no. 4, 25 pp.

[2] J.L. Loday, M. Ronco (2006). On the structure of cofree Hopf algebras. *J. Reine Angew. Math.* 592, 123–155.

[3] M. Markl (2015). On the origin of higher braces and higher-order derivations. *J. Homotopy Relat. Struct.* 10, no. 3, 637–667.

Associativity or commutativity, but not both

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Resumen. Given a simplicial set X, there is a well-known operation on the normalized cochain complex $N^*(X)$: the cup product. It is associative but, in general, not graded commutative (it becomes so in cohomology, but not at the cochain level). This failure of commutativity is measured by the Steenrod operations.
On the other hand, working over the field of rational numbers and using Sullivan's polynomial differential forms on simplices together with Dupont's contraction, we can transfer a C_∞-structure onto $N^*(X)$. Considering the operation of arity 2, we obtain a commutative but not associative multiplication.
The relationship between these two operations is quite mysterious (at least to me!). However, we do know that they coincide in cohomology, where we recover the usual commutative and associative cup product. In this talk, containing more questions than answers, we will present these structures and some facts about them.

Realizing finite subsets of $\mathbb{Q}$ as mapping degree sets

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Resumen. Given two oriented closed connected manifolds M and N of the same dimension, the mapping degree set is

$$deg(M,N) = \{deg(f) \mid \ f\colon M \to N \text{ continuous}\}.$$

A central problem is to determine which subsets of integers (containing 0) can occur as mapping degree sets. By a simple cardinality argument, not every infinite subset of $\mathbb{Z}$ can occur, while the full picture remains an open problem.
In this talk we will discuss the rational version: using Sullivan models, we prove that every finite subset of $\mathbb{Q}$ (containing 0) can be realized as the mapping degree set of oriented closed simply-connected manifolds. The techniques we develop provide key tools to address the integral case, where the same affirmative result holds in the finite setting.
This is a joint work with Vicente Muñoz and Antonio Viruel.

Teoría de homotopía estable racional

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Resumen. Es bien sabido que la categoría homotópica estable racional no es más que la categoría homotópica de los complejos de cadena.
Extendemos este resultado a la teoría de homotopía estable racional parametrizada que a su vez transcurre a lo largo de otras tantas categorías de espectros de categorías algebraicas.
Esta charla está basada en trabajo conjunto con Alejandro Saiz.

Estructuras en geometría no-conmutativa

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Resumen. Estudiamos diferentes maneras de obtener estructuras en geometría no-conmutativa, focalizando en versiones no-conmutativas de las estructuras de Poisson, bi-Hamiltonianas y Poisson-Nijenhius.

The algebraic structures of social organizations: the operad of cooperative games

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Resumen. Cooperative game theory studies how cooperation arises between individuals depending on the eventual gains that each possible coalition can obtain. This is modelled by the notion of a cooperative game, which amounts to the data of a set of players N and a set function from the power set P(N) to the real numbers. The main goal of this talk is to explain how one can study cooperative games using operad theory. We start by endowing the collection of all cooperative games with any number of players with an operad structure. Then, we will explain how the composition in this operad generalizes to the notions composition/aggregation of games considered before by Owen, Shapley, von Neumann and Morgenstern, and many others. Furthermore, we explicitly compute this operad in terms of generators and relations, and show that it corresponds via the Möbius transform to the operad encoding commutative triassociative algebras. If time permits, we will then explain how this operad structure restricts to many interesting classes of games (simple, balanced, capacities a.k.a fuzzy measures and convex functions, totally monotone, etc) and how one can obtain a new operad structure on the family of all generalized permutahedra using this. Finally, we will explain how different solution concepts such as the core and the Shapley or the Banzhaf values interact with the composition of this operad.
This is joint work with Dylan Laplace Mermoud, https://arxiv.org/abs/2507.01969.

Unos productos de Massey sensibles a la estructura conmutativa en términos de determinantes

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Resumen. Presentamos unas operaciones secundarias nuevas en la (co)homología de una cdga (álgebra graduada diferencial conmutativa) y de algunas álgebras con condiciones de asociatividad/conmutatividad más débiles. Estos nuevos productos están relacionados con los productos de Massey matriciales y los determinantes, y permiten detectar sutilezas respecto a los distintos niveles de conmutatividad que puede tener un álgebra n-asociativa. Este es un trabajo en progreso con Oisín Flynn-Conolly y Fernando Muro.