Evolution PDEs in applied sciences

Equipo organizador

- Alejandro Gárriz (Universidad de Granada)
- David Poyato (Universidad de Granada)
- Fernando Quirós (Universidad Autónoma de Madrid)

Descripción

La sesión se centra en el análisis matemático de una amplia clase de ecuaciones en derivadas parciales (EDPs) que describen diversos fenómenos en física y biología. Desde un punto de vista matemático, estas EDPs son a menudo ecuaciones integrodiferenciales hiperbólicas o parabólicas con términos no locales y no lineales que hacen el análisis complejo, tanto desde el punto de vista teórico como computacional. Entre los fenómenos de interés que se abordarán en la sesión destacan: coagulación-fragmentación, agregación-difusión, reacción-difusión, dinámica colectiva, quimotaxis, difusión no lineal y/o no local, formación de patrones, dinámica tumoral, etc.

Algunas de los cuestiones matemáticos que se abordarán en relación con dichos fenómenos son: existencia, unicidad, regularidad y estabilidad de las soluciones, existencia global vs explosión en tiempo finito, comportamiento a largo plazo, comportamiento a largo plazo, límites de la escala microscópica a la macroscópica, flujos gradiente, transporte óptimo de masa, controlabilidad y control óptimo, problemas inversos, etc.

Palabras clave: Ecuaciones en Derivadas Parciales; Física; Biología.

Programa

Jueves, 22 de enero

11:00 – 11:30	José M. Mazón (Universidad de Valencia) *Evolution problems with perturbed 1-Laplacian type operators on random walk spaces*
11:30 – 12:00	Félix del Teso (Universidad Autónoma de Madrid) *Mean Value Properties for Local and Nonlocal p-Laplace Problems*
12:00 – 12:30	Inmaculada Benítez (Universidad de Granada) *Asymptotic behaviour of solutions to the Becker-Döring equations*
12:30 – 13:00	Juan Calvo (Universidad de Granada) *The Lifshitz-Slyozov system with inflow boudary conditions*
16:00 – 16:30	Tomás Alarcón (ICREA, CRM) *Particle simulations of the interaction between cancer and immune cells*
16:30 – 17:00	Martina Conte (Politecnico di Torino) *Modeling cell dynamics across multiple scales*
17:00 – 17:30	Mingmin Zhang (University of Science and Technology of China) *Front propagation for a transport model with a nonlocal condition of the Fisher-KPP type at the boundary*
18:00 – 18:30	María A. Rodriguez Bellido (Universidad de Sevilla) *Looking for an adequate framework for studying optimal control problems restricted to chemotaxis PDE models*
18:30 – 19:00	Martina Magliocca (Universidad de Sevilla) *Traveling Motility of Actin Lamellar Fragments Under spontaneous symmetry breaking*

Viernes, 23 de enero

11:00 – 11:30	Antonio Suárez (Universidad de Sevilla) *Lotka-Volterra models with nonlocal coefficient diffusion*
11:30 – 12:00	María José Cáceres (Universidad de Granada) *Asymtotic behaviour of some mesoscopic models for neurons populations*
12:00 – 12:30	José A. Cañizo (Universidad de Granada) *Diffusive behaviour of some linear kinetic equations*

Evolution problems with perturbed 1-Laplacian type operators on random walk spaces

José M. Mazón

Departamento de Análisis Matemático, Universidad de Valencia

mazon@uv.es

Resumen. Random walk spaces are a general framework for the study of PDEs. They include as particular cases locally finite weighted connected graphs and nonlocal settings involving symmetric integrable kernels on RN. We are interested in the study of evolution problems involving two random walk structures so that the associated functionals have different growth on each structure. We also deal with the case of a functional with different growth on a partition of the random walk.
Joint work with W. Górny and J. Toledo.

Referencias

[1] W. Górny, J. M. Mazón, J. Toledo (2025). Evolution problems with perturbed 1-Laplacian type operators on random walk spaces. *Mathematische Annalen*.

Mean Value Properties for Local and Nonlocal p-Laplace Problems

Félix del Teso

Departamento de Matemáticas, Universidad Autónoma de Madrid

felix.delteso@uam.es

Resumen. The aim of this talk is to present recent advances on asymptotic expansions, mean value properties, and finite difference schemes for parabolic and elliptic equations involving the p-Laplacian and fractional p-Laplacian operators.

Asymptotic behaviour of solutions to the Becker-Döring equations

Inmaculada Benítez

Departamento de Matemática Aplicada, Universidad de Granada

inmbenber@ugr.es

Resumen. Coagulation and fragmentation equations are mathematical models that describe the processes of particle aggregation (coagulation) and division (fragmentation) within a population. These equations, utilized in fields such as chemistry, physics, and biology, are crucial for understanding the dynamics of particle systems. Coagulation equations depict how particles merge to form larger entities, while fragmentation equations illustrate the breakdown of larger particles into smaller ones.

The Becker-Döring equations, introduced by Becker and Döring (1935), serve as fundamental frameworks for describing first-order phase transitions in various chemical and physical systems, such as vapor condensation and lipid aggregation. Over the years, significant attention has been devoted to studying this model in the nonlinear case in contrast to the linear situation, such as the work by Penrose (1989) where metastable states were studied, or Cañizo & Lods (2013), studying the exponential convergence of the solution only in the subcritical case. For this reason, we focus on the linear Becker-Döring equations, defined by:

$$\frac{d}{dt}c_i(t) = W_{i-1} - W_i, \quad \text{for all } i \geq 2$$

being c_1 constant for all $t \in \mathbb{R}$, where the fluxes W_i depend on the coagulation and fragmentation coefficients a_i and b_i, respectively, following the expression $W_i = a_i c_1 c_i - b_{i+1} c_{i+1}$.
Our aim is to examine the long-term behaviour of solutions to the equilibria, providing a more accurate estimation of the convergence rate than that shown by Kreer (1993). To achieve this, we employ spectral theory and entropy methods, using the free energy provided also by Kreer (1993).

The Lifshitz-Slyozov system with inflow boudary conditions

Juan Calvo

Departamento de Matemática Aplicada, Universidad de Granada

juancalvo@ugr.es

Resumen. The Lifshitz-Slyozov system describes the temporal evolutions of a mixture of monomers and aggregates, where monomers can attach to or detach from already existing aggregates. The aggregate distribution follows a transport equation with respect to a size variable, whose transport rates are coupled to the dynamics of monomers in a nonlocal fashion. Recent applications to protein polimerization phenomena introduce attachment and detachment rates that require a nonlinear boundary condition at zero size, which describes nucleation processes. In this talk we review and discuss the available results for this model concerning well-posedness and long-time dynamics, with a focus on those cases where concentration at small sizes is observed.

Referencias

[1] J. Calvo, E. Hingant, R. Yvinec (2021). The initial-boundary value problem for the Lifshitz-Slyozov equation with non-smooth rates at the boundary. *Nonlinearity*, 34, 1975–2017.

[2] J. Calvo, E. Hingant, R. Yvinec (2022). Some remarks about the well-posedness of Lifshitz-Slyozov's equations with nucleation kinetics. To appear in *Proceedings of the International Conference on Hyperbolic problems (HYP22)*.

[3] J. Calvo, E. Hingant, R. Yvinec (2024). Long-time asymptotic of the Lifshitz-Slyozov equation with nucleation. *Kinetic and Related Models*, 17, 755–773.

Individual-based model of cancer-immune system interactions: The role of cell adhesion and immune cell exhaustion

Tomás Alarcón

ICREA, CRM

alarcon@crm.cat

Resumen. We present a particle-based, individual based model of the interactions between cancer and immune cells "in vitro". We have formulated a new off-lattice individual-based model (IBM) which extends existing models of immune-system infiltration into tumours by accounting for dynamic variation in the exhaustion of the immune cells and damage incurred by the tumour cells. The IBM model distinguishes two types of cells: immune cells and cancer cells. Their dynamics are influenced by two internal variables (structure). In the case of cancer cells, the internal variable accounts for the number of encounters or "hits" each of them has had with the immune cells. Immune cells are characterised by a rate of exhaustion (RE), which accounts for how fast immune cells lose cytolytic activity as they accumulate encounters with the cancer population until eventually they cannot carry out their anti-cancer function any longer. In this work, we investigate how cell adhesion molecules (such as CD2 and ICAM-1) affect immune cell infiltration into cluster of tumour cells and how immune cell exhaustion is affected by upregulation of such molecules.

Modeling cell dynamics across multiple scales

Martina Conte

Department of Mathematical Sciences "G.L. Lagrange", Politecnico di Torino

martina.conte@polito.it

Resumen. The characterization of biological phenomena related to cell evolution and their interactions with the microenvironment often involves processes occurring across a range of spatial and temporal scales. As a result, mathematical models designed to describe cell dynamics must capture this inherent multiscale complexity. In this talk, we introduce a multiscale mathematical framework based on kinetic equations to describe cellular mechanisms that are well characterized at different scales—from the individual level, where stochastic mechanisms of cell reorientation can be incorporated, to the kinetic level, which describes collective cell motion in response to microenvironmental stimuli, and ultimately to macroscopic descriptions of cellular aggregates.

Front propagation for a transport model with a nonlocal condition of the Fisher-KPP type at the boundary

Mingmin Zhang

School of Mathematical Sciences, University of Science and Technology of China

mingmin.zhang.math@gmail.com

Resumen. The model under study is a linear transport equation in the upper half plane, together with a nonlinear and nonlocal Dirichlet condition that couples the values of the unknown function at the boundary to those inside. The objective is to understand its precise asymptotics in time. Its structure is reminiscent of that of the "Road-field model" introduced by Berestycki, Roquejoffre and Rossi. Its primary motivation is the study of the nonlocal Kermack-McKendrick model for the spread of epidemics, and it presents specific issues calling for a mathematical study of its own. I will talk about the derivation of the model and discuss some of its general properties. Then, I will show that any initially localized solution will lag behind the minimal traveling waves and the delay grows logarithmically in time. This is a joint work with G. Faye and J.-M. Roquejoffre.

Referencias

[1] G. Faye, J.-M. Roquejoffre, M. Zhan (2023). Spreading properties in Kermack - McKendrick models with nonlocal spatial interactions – A new look. *hal-04078812*

Looking for an adequate framework for studying optimal control problems restricted to chemotaxis PDE models

María A. Rodríguez Bellido

Departamento de Ecuaciones Diferenciales y Análisis Numérico, IMUS, Universidad de Sevilla

angeles@us.es

Resumen. Mathematical models of chemotaxis attempt to reproduce biological processes in which spatial movement of a living organism is modified by the presence of a chemical substance (see [1] for a review). The biological importance of these processes and the advantages of a mathematical approach from an ethical point of view are some of the reasons why the mathematical literature on this topic is so extensive. No less important are the attempts to control the behavior of the variables involved to achieve the desired effects. However, these models are not very regular, mathematically speaking, which makes their analysis difficult.

In this work we address the formulation of control problems restricted to some chemotaxis models and we try to establish an optimal functional framework to deduce rigorously optimality conditions for the optimal control problems.

The results presented in this talk are mainly based on [2, 3, 4, 5].

Referencias

[1] N. Bellomo, A. Bellouquid, Y. Tao, M. Winkler (2015). Toward a mathematical theory of Keller-Segel models of pattern formation in biological tissues. *Math. Models Methods Appl. Sci.* 25(9), 1663–1763.

[2] P. Braz e Silva, F. Guillén-González, C. F. Perusato, M. A. Rodríguez-Bellido (2023). Bilinear optimal control for weak solutions of the Keller-Segel logistic model in 2D domains. *Appl. Math. Optim.* 87, article number 55.

[3] F. Guillén-González, E. Mallea-Zepeda, M. A. Rodríguez-Bellido (2020). Optimal bilinear control problem related to a chemo-repulsion system in 2D domains. *AESAIM Control Optim. Calc. Var.* 25, article number 29 article number 55.

[4] F. Guillén-González, E. Mallea-Zepeda, M. A. Rodríguez-Bellido (2020). A Regularity Criterion for a 3D Chemo-Repulsion System and Its Application to a Bilinear Optimal Control Problem. *SIAM J. Control and Optimization* 58(3), 1457–1490.

[5] F. Guillén-González, E. Mallea-Zepeda, M. A. Rodríguez-Bellido, E. J. Villamizar-Roa (2025). Optimal bilinear control restricted to the three-dimensional chemo-repulsion model with potential production. *Work in progress.*

Traveling Motility of Actin Lamellar Fragments Under spontaneous symmetry breaking

Martina Magliocca

Departamento de Análisis Matemático, Universidad de Sevilla

mmagliocca@us.es

Resumen. Cell motility is connected to the spontaneous symmetry breaking of a circular shape. In [1], Blanch-Mercader and Casademunt perfomed a nonlinear analysis of the minimal model proposed by Callan and Jones in [2] and numerically conjectured the existence of traveling solutions once that symmetry is broken. In this work, we prove analytically that conjecture by means of nonlinear bifurcation techniques.

These results have been obtained in collaboration with Claudia García, Martina Magliocca, Nicolas Meunier.

Referencias

[1] C. Blanch-Mercader, J. Casademunt (2013). Spontaneous motility of actin lamellar fragments. *Physical review letters*, 110(7), 078102.

[2] A. C. Callan-Jones, J. F. Joanny, J. Prost (2013). Viscous-fingering-like instability of cell fragments. *Physical review letters*, 100(25), 258106.

Lotka-Volterra models with nonlocal coefficient diffusion

Antonio Suárez

Universidad de Sevilla

suarez@us.es

Resumen. In this talk we present theoretical results of the Lotka-Volterra models with non-local diffusion, specifically, the diffusion coefficients depend on the total population in a nonlinear way. This kind of diffusion models that the species tends to leave crowded areas or is attracted to regions with higher population density, depending on whether the nonlinear function increases or decreases, respectively. The inclusion of these non-local terms in the diffusion coefficients entails significant technical difficulties. We show results of the existence and non-existence of coexistence states of the models depending on the coefficients of the model

Asymtotic behaviour of some mesoscopic models for neurons populations

María J. Cáceres

Departamento de Matemática Aplicada, Universidad de Granada

caceresg@ugr.es

Resumen. In recent decades, the mathematical science community has made significant contributions to the study of neuronal population dynamics, proposing a wide variety of models. Among these are two families based on partial differential equations: the NNLIF model (Nonlinear Noisy Leaky Integrate and Fire) and the time-since-last-discharge model. The former is grounded in a nonlinear Fokker–Planck type equation, while the latter is described by a nonlinear age-structured equation. In the NNLIF model, the neuronal population is represented in terms of the membrane potential, whereas in the age-structured model, the variable is the time elapsed since the last discharge.

In this talk, we will study the asymptotic behavior of these models. Different techniques have been employed for this purpose. One of them, widely used in kinetic theory, is the entropy dissipation method.

We will discuss the limitations of this approach for these models and present alternative techniques based on two main ideas: 1) the analysis of a much simpler discrete-type model, and 2) the reformulation of the problem, via the Laplace transform, as a Volterra-type integral equation.

This talk is based on works in collaboration with José A. Cañizo, Alejandro Ramos-Lora and Nicolás Torres.

Diffusive behaviour of some linear kinetic equations

José A. Cañizo

Departamento de Matemática Aplicada, Universidad de Granada

canizo@ugr.es

Resumen. We consider linear kinetic equations of the form $\partial_t f + \frac{1}{\epsilon} v \nabla_x f = \frac{1}{\epsilon^2} L(f)$, for an unknown f which depends on time t, position x and velocity v, and where L is a linear operator which acts only in the velocity variable, and which typically has a probability equilibrium in v. Important examples include the Fokker-Planck operator, nonlocal diffusion operators, linear BGK-type operators, or linear Boltzmann operators. This PDE typically represents a mesoscopic physical model, where we keep track of the probability distribution of the position and velocity of particles. It is well known that when ϵ tends to 0, this type of equation has a macroscopic or diffusive limit for the density $\rho(t, x) := \int f(t, x, v) dv$, which is either the standard heat equation, or the fractional heat equation. As a new result, we show that for a fixed epsilon, the behaviour of this equation for large times also follows the standard or fractional heat equation, and that the long-time and small-epsilon limits are actually interchangeable in many cases. This is a work in collaboration with Stéphane Mischler (U. Paris-Dauphine) and Niccolò Tassi (U. Granada).