Fluid Dynamics and PDEs

Equipo organizador

- Eduardo García-Juárez (Universidad de Sevilla)
- Antonio Hidalgo-Torné (Max Planck Institute for Mathematics in the Sciences)
- Fran Mengual (Universidad de Sevilla)
- Elena Salguero (Max Planck Institute for Mathematics in the Sciences)

Descripción

The study of fluid dynamics plays a crucial role in mathematics and finds multiple applications in the natural sciences. The aim of this session is to bring together outstanding researchers who will present recent developments in the theoretical understanding of partial differential equations (PDEs) arising from fluid dynamics and related topics.

Key topics include analytical challenges such as the existence and uniqueness of solutions, stability properties, singularity formation, bifurcation analysis, and the asymptotic behavior of fluid models. The session will focus on cutting-edge research in the analysis of PDEs in fluid mechanics and provide a platform for collaboration and the exchange of ideas among experts in the field.

Palabras clave: Fluid mechanics; Partial differential equations; Turbulence; Stability

Programa

Lunes, 19 de enero

15:30 – 16:00	Diego Alonso-Orán (Universidad de La Laguna) *Rotating solutions to the incompressible Euler-Poincaré equation with external particle*
16:00 – 16:30	Camilla Nobili (University of Surrey) *Enhanced Dissipation via Time-Dependent Velocity Fields*
16:30 – 17:00	Ángel Castro (ICMAT) *Travelling and rotating solutions close to shear flows*
17:00 – 17:30	Michele Dolce (EPFL) *Long wave instabilities of general shear flows for 2D viscous fluids*

Martes, 20 de enero

11:00 – 11:30	Clara Patriarca (Université Libre de Bruxelles) *Stability of equilibria and bifurcations for a fluid-solid interaction problem*
11:30 – 12:00	Francisco Gancedo (Universidad de Sevilla) *2D Navier-Stokes free boundary: nonnegative density and viscosity contrast*
12:00 – 12:30	Gonzalo Cao-Labora (EPFL) *Computer-assisted techniques for proving stability in PDEs*
15:30 – 16:00	Francesco Fanelli (BCAM) *Geometric traits in the incompressible Euler equations with variable density*
16:00 – 16:30	Zineb Hassainia (Universidad de Granada) *Leapfrogging Motion in Fluid Dynamics*
16:30 – 17:00	Joan Mateu (Universitat Autònoma de Barcelona) *On the analyticity of the trajectories of the particles in the patch problem for some active scalar equations*
17:00 – 17:30	Annalaura Rebucci (Max Planck Institute MiS) *On the De Giorgi-Nash-Moser regularity theory for kinetic hypoelliptic operators*
18:00 – 18:30	Edoardo Bocchi (Politecnico di Milano) *Asymptotics of the Stokes problem close to solid contacts*
18:30 – 19:00	Rafael Granero (Universidad de Cantabria) *On the mathematical description of tumor growth*

Rotating solutions to the incompressible Euler-Poincaré equation with external particle

Diego Alonso-Orán

Universidad de La Laguna

dalonsor@ull.edu.es

Resumen. In this talk, we consider a two-dimensional, incompressible fluid body, together with self-induced interactions. The body is perturbed by an external particle with small mass. The combined fluid–particle system rotates uniformly about its common center of mass. Using perturbative methods, we construct solutions that are stationary in a uniformly rotating frame. In addition, we identify a broad class of internal fluid motions compatible with this regime. If time permits, we will also outline forthcoming results.

Enhanced Dissipation via Time-Dependent Velocity Fields

J. Benthaus, G. M. Coclite, Camilla Nobili

University of Surrey

c.nobili@surrey.ac.uk

Resumen. Enhanced dissipation is a phenomenon of central importance in both physics and mathematics, owing to its fundamental role in fluid mixing and its wide-ranging applications, particularly in chemical processes. Over the past two decades, major advances in mathematical fluid mechanics have yielded rigorous descriptions of this effect, including precise quantitative estimates of energy dissipation rates for specific classes of flows (Constantin, Kiselev, Ryzhik & Zlatos 2008; Bedrossian & Coti Zelati 2017; Coti Zelati, Delgadino & Elgindi 2019). Nonetheless, for time-dependent velocity fields, the inherent dynamical complexity and the limitations of current analytical techniques make obtaining quantitative results significantly more challenging, and this area remains comparatively less developed.

In this talk, I will focus on certain classes of time-dependent shear flows and present mixing estimates together with quantitative bounds that display (super-)enhanced dissipation. Some of these results draw inspiration from mixing mechanisms commonly employed in analytical laboratories, offering new insights into how particular flow structures can dramatically accelerate dissipation.

This is based on recent joint works with Johannes Benthaus and Giuseppe Maria Coclite.

Travelling and rotating solutions close to shear flows

Ángel Castro, D. Lear

ICMAT

angelcastro@icmat.es

Resumen. In this talk we will consider the existence of rotating solutions arbitrarily close (in some topology) to radial monotone decreasing vorticity for 2D Euler. In a paper by Bedrossian, Coti-Zelati and Vicol was shown that radial monotone decreasing vorticities are stable at the linear level, thus, our result shows that this phenomenon can break even for small perturbation. The problem is related with the stability of shear flows and the existence of stationary and traveling waves solution near them. We also review some results on this topic. This is a joint work with Daniel Lear.

Long wave instabilities of general shear flows for 2D viscous fluids

M. Colombo, Michele Dolce, R. Montalto, P. Ventura

EPFL

michele.dolce@epfl.ch

Resumen. In 1959, Kolmogorov proposed to study the instability of the shear flow (sin(y),0) in the vanishing viscosity regime in tori of different aspect ratios. This question was later resolved by Meshalkin and Sinai in the '60s. Generalizing their picture, we focus on instability properties for general shear flows (U(y),0) and we show that they always exhibit a long wave instability mechanism. This confirms previous findings by Yudovich in 1966 and is established through two independent approaches: one via the construction of Kato's isomorphism and one via normal forms. In both cases, unlike in many other applications of these methods, the corresponding operators are not small perturbations of a given simpler operator. This is a joint work with M. Colombo, P. Ventura and R. Montalto.

Stability of equilibria and bifurcations for a fluid-solid interaction problem

D. Bonheure, G. P. Galdi, Clara Patriarca

Université Libre de Bruxelles

clara.patriarca@ulb.be

Resumen. We study certain significant properties of the equilibrium configurations of a coupled system S, consisting of an incompressible Navier Stokes fluid in an unbounded 3D domain interacting with a structure formed by a rigid body subject to undamped elastic restoring forces and torque around its rotation axis. The motion of S is driven by a uniform flow at spatial infinity, with constant dimensionless velocity λ. We show that if λ is below a critical value, λc (say), there is a unique and stable time-independent configuration, where the body is in equilibrium and the flow is steady. Successively, we investigate possible loss of uniqueness by providing necessary and sufficient conditions for the occurrence of a steady bifurcation at some $\lambda s \geq \lambda c$. This is a joint work with D. Bonheure (Université Libre de Bruxelles) and G.P. Galdi (University of Pittsburgh).

2D Navier-Stokes free boundary: nonnegative density and viscosity contrast

Francisco Gancedo

Universidad de Sevilla

fgancedo@us.es

Resumen. This talk concerns the evolution of two incompressible, immiscible fluids in two dimensions, governed by the inhomogeneous Navier-Stokes equations. We present new global-in-time well-posedness results, showing the preservation of the natural $C^{1+\gamma}$ Hölder regularity of the moving interface, for $0 < \gamma < 1$. This is the first approach that allows for nonnegative density driven by a low-regularity initial velocity, while also remaining valid in the presence of a small viscosity jump.

Computer-assisted techniques for proving stability in PDEs

Gonzalo Cao-Labora

EPFL

gonzalo.caolabora@epfl.ch

Resumen. This talk will explain how computer-assisted techniques can be used to prove stability (possibly up to a finite codimension) of PDE solutions. A key application is to demonstrate the existence of exact smooth self-similar solutions near numerically obtained approximations. This provides a mechanism to rigorously prove singularity formation without forcing, starting from a numerically computed profile. Other applications include the stability of blow-up formation for NLS equations and the stability of solitons. We will provide a brief overview of the problems where this technique is useful and then focus on a detailed application to a “toy linear operator”, going carefully through the mathematical arguments and outlining the code implementing the computer-assisted part.

Geometric traits in the incompressible Euler equations with variable density

Francesco Fanelli

BCAM

ffanelli@bcamath.org

Resumen. This talk focuses on the analysis of the incompressible Euler equations with variable density. Their homogeneous counterpart, which is given by the classical incompressible Euler system, is well-known to be globally well-posed in the case of space dimension $d = 2$, both in the strong solutions framework and in the Yudovich (weak) solutions framework. No results of that kind are known in the case of density variations.
In this talk, we will show that both problems (that is, global well-posedness and weak solutions theory *à la Yudovich*) can be reduced to the study of a non-linear geometric quantity, which encodes the regularity of the velocity field along the level lines of the density.

Leapfrogging Motion in Fluid Dynamics

Zineb Hassainia

Universidad de Granada

zinebhassainia@ugr.es

Resumen. In this talk, I will present recent results on the leapfrogging phenomenon in vortex dynamics for the Euler equations. We prove that, under suitable constraints, concentrated vortex patches exhibit leapfrogging behavior for all time. When viewed from a translating frame of reference, the dynamics of these patches correspond to a non-rigid, time-periodic motion. The analysis relies on KAM theory, which allows us to overcome the small divisors problem and to address the degeneracy in the time direction.

On the analyticity of the trajectories of the particles in the patch problem for some active scalar equations

Joan Mateu

Universitat Autònoma de Barcelona

joan.mateu@uab.cat

Resumen. Let Ω be a bounded domain in R^n whose boundary is $C^{1,\gamma}$ for $\gamma \in (0,1)$. Consider 2D Euler equation for the vorticity or the n-D aggregation equation in the case of the initial condition being a positive multiple of the characteristic function of Ω. In this talk we discuss on global in time analyticity of the flow generated by the velocity field which propagates the vorticity or density solution respectively. These results are obtained from a detailed study of the Beurling or Riesz transform, that represents derivatives of the velocity field. The precise estimates obtained for the solutions of an equation satisfied by the Lagrangian flow, are a key point in the development.

On the De Giorgi-Nash-Moser regularity theory for kinetic hypoelliptic operators

F. Anceschi, H. Dietert, J. Guerand, A. Loher, C. Mouhot, Annalaura Rebucci

Max Planck Institute for Mathematics in the Sciences

annalaura.rebucci@mis.mpg.de

Resumen. We present new results that extend the De Giorgi-Nash-Moser theory to a class of hypoelliptic equations naturally arising in kinetic theory. These operators are mathematically characterized by two parts: a diffusion part governed by the (fractional) Laplace operator in some set of variables and a first order operator that contains the directions of missing ellipticity. A key ingredient to prove our results is a Poincaré inequality, which we derive from the construction of suitable trajectories. The trajectories we rely on are quite flexible and allow us to consider equations with an arbitrary number of Hörmander commutators and whose diffusive part is either local (second-order) or nonlocal (fractional order). We later combine the Poincaré inequality with a $L^2 - L^\infty$ estimate, a Log-transformation and a classical covering argument (Ink-Spots Theorem) to deduce Harnack inequalities and Hölder regularity along the line of De Giorgi method.
This talk is based on a series of papers in collaboration with F. Anceschi, H. Dietert, J. Guerand, A. Loher and C. Mouhot.

Asymptotics of the Stokes problem close to solid contacts

Edoardo Bocchi, D. Bonheure, M. Hillairet

Politecnico di Milano

edoardo.bocchi@polimi.it

Resumen. In the field of fluid-solid interactions, a crucial question is the description of the flow of a fluid confined between moving solid surfaces, either elastic or rigid. In this context, it is usual to consider the stationary Stokes problem, supplemented with boundary conditions that encode the motion of the surfaces. In this talk, I will present a variational method that allows to compute and rigorously justify the asymptotic expansion of the Stokes solution when the surfaces are close to isolated contacts. This talk is based on a joint work with D. Bonheure and M. Hillairet.

On the mathematical description of tumor growth

Rafael Granero

Universidad de Cantabria

rafael.granero@unican.es

Resumen. In this talk we will present some new results and models on the mathematical description of multilayer tumor growth.