Geometría Algebraica Aplicada y Computacional

Equipo organizador

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Descripción

Muchos problemas en aplicaciones, como las redes de reacciones químicas, teoría del contro y estadística entre otras muchas, se reducen a problemas con polinomios, esto es, problemas computacionales en geometría algebraica. Entender cómo resolver estos problemas de un modo eficiente y confiable es esencial para afrontar los desafíos en estas aplicaciones. En esta sesión, investigadores en geometría algebraica aplicada y computacional hablarán sobre los últimos desarrollos en este intercambio entre aplicaciones, geometría algebraica y computación.

Palabras clave: aplicaciones, computación, polinomios, geometría algebraica.

Programa

Lunes, 19 de enero

15:30 – 16:00	Elisenda Feliu (University of Copenhagen) *The Generic Geometry of Steady State Varieties of Reaction Networks*
16:00 – 16:30	Luca Sodomaco (MPI MiS, Leipzig) *Higher-order Osculating Eigenvectors of Symmetric Tensors*
16:30 – 17:00	Elvira Pérez-Callejo (Universidad de Valladolid) *An effective algorithm for algebraic integrability of polynomial foliations with bounded genus*
17:00 – 17:30	Simon Telen (MPI MiS, Leipzig) *Toric amplitudes and universal adjoints*

Martes, 20 de enero

11:00 – 11:30	Carlos D'Andrea (Universitat de Barcelona) *Effective Pourchet's Theorem*
11:30 – 12:00	Carles Checa (University of Copenhagen) *Effective criteria for birrationality of biquadratic polynomial maps*
12:00 – 12:30	Javier Sendra Arranz (CUNEF University) *Hilbert schemes of points of singular curves*
15:30 – 16:00	Marta Casanellas (Universitat Politècnica de Catalunya) *Distinguishing phylogenetic networks*
16:00 – 16:30	Roser Homs Pons (Universitat Politècnica de Catalunya) *Maximum likelihood thresholds for colored Gaussian graphical models*
16:30 – 17:00	Angel David Rios Ortiz (Université Paris-Saclay) *Crossing the Transcendental Divide: From Schottky Groups to Algebraic Curves*
17:00 – 17:30	Mario González-Sánchez (Universidad de Valladolid) *Some advances on the Eisenbud-Goto conjecture for projective toric surfaces*
18:00 – 18:30	Eduardo Sáenz de Cabezón (Universidad de La Rioja) *Involutively stable ideals*
18:30 – 19:00	Chiara Meroni (ETH Institute for Theoretical Studies) *Rank-one convexity*

The Generic Geometry of Steady State Varieties of Reaction Networks

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Resumen. The mathematical formalism of the study of (bio)chemical reaction networks goes back to at least the works of Feinberg, Horn and Jackson in the 70ies. However, several fundamental questions about the positive part of the algebraic varieties defined by the steady state equations remained unsolved. In this talk, I will describe several properties that these semialgebraic sets display generically with respect to the parameter values. The main message is that the generic geometries of the semialgebraic sets and the complex algebraic variety agree, to a great extent. I will address in particular generic finiteness of steady states, nondegeneracy, toricity, absolute concentration robustness, and the nondegenerate multistationarity conjecture.

Referencias

[1] E. Feliu, O. Henriksson, B. Pascual-Escudero (2025). Generic consistency and nondegeneracy of vertically parametrized systems. *Journal of Algebra* 677, 630–666.

[2] E. Feliu, O. Henriksson, B. Pascual-Escudero (2024+). The generic geometry of steady state varieties. arXiv:2412.17798

Higher-order Osculating Eigenvectors of Symmetric Tensors

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Resumen. Consider the space of real n-ary symmetric tensors of order d, equipped with the Bombieri-Weyl inner product. Let X be the affine cone of real symmetric tensors of rank at most one. For a fixed real symmetric tensor f, we study the distance function $d_{f,X}$ from f restricted to X. When f is generic, the number of complex critical points of $d_{f,X}$ is finite and constant, and coincides with the Euclidean Distance degree of X with respect to the Bombieri-Weyl inner product.
In this talk, we describe the locus of symmetric tensors f with at least one critical point x of $d_{f,X}$ which is k-osculating for some integer $k > 1$, meaning that $f - x$ is orthogonal to the k-th osculating space of X at x. This leads to the notion of k-osculating eigenvector of f.

An effective algorithm for algebraic integrability of polynomial foliations with bounded genus

Elvira Pérez-Callejo, Carlos Galindo, Francisco Monserrat

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Resumen. Characterizing algebraic integrability of planar polynomial foliations is a classical and open problem. This talk presents an algorithm for effective algebraic integrability using extensions to rational surfaces. It decides whether a polynomial foliation on the complex plane possesses a rational first integral of given genus $g \neq 1$, and computes this integral if it exists.
The method relies on geometrical objects obtained from the dicritical reduction of singularities of the extended foliation, specifically a certain $\mathbb{Q}$-divisor $T_{\mathcal{F}}$ on the resulting surface.

Referencias

[1] C. Galindo, F. Monserrat, E. Pérez-Callejo (2025). Algebraic integrability with bounded genus. *Journal of Differential Equations* 440 (Part 2), 113466.

Toric amplitudes and universal adjoints

Simon Telen

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Resumen. A toric amplitude is a rational function associated to a simplicial polyhedral fan. The definition is inspired by scattering amplitudes in particle physics. We prove algebraic properties of such amplitudes and study the geometry of their zero loci. These hypersurfaces play the role of Warren's adjoint via a dual volume interpretation. We investigate their Fano schemes and singular loci via the nef cone and toric irrelevant ideal of the fan.

Referencias

[1] S. Telen (2025+). Toric Amplitudes and Universal Adjoints. arXiv:2504.00897

Effective Pourchet's Theorem

Carlos D'Andrea, Teresa Cortadellas Benítez, Ana Belén de Felipe, Joel Hurtado, Eulàlia Montoro

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Resumen.It has been proven by Y. Pourchet in [2] that any positive polynomial $f(x) \in \mathbb{Q}[x]$ can be written as a sum of 5 squares of polynomials in $\mathbb{Q}[x]$, and that the number 5 is optimal in the sense that there are positive polynomials which cannot be written as a sum of 4 or less squares.
Recently, an algorithm was proposed in [1] with a conjecture that it may always produce an optimal decomposition as a sum of 5 squares.
In this presentation, we will review the problem and its algorithmic approach, and show that the conjecture in [1] is not correct. We also explore further generalizations of the results given there.

Referencias

[1] V. Magron, P. Koprowski, T. Vaccon (2023). Pourchet's theorem in action: decomposing univariate nonnegative polynomials as sums of five squares, en *Proceedings of the International Symposium on Symbolic & Algebraic Computation (ISSAC 2023)*, ACM, 425–433.

[2] Y. Pourchet (1971). Sur la représentation en somme de carrés des polynômes à une indéterminée sur un corps de nombres algébriques. *Acta Arith.* 19, 89–104.

Effective criteria for birrationality of biquadratic polynomial maps

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Resumen. A biquadratic rational map is a map $\mathbb{P}^1 \times \mathbb{P}^1 \dashrightarrow \mathbb{P}^2$ whose entries are biquadratic polynomials. Some of the algebraic aspects related to determining when these type of maps are birational had been studied, specially with a view towards applications in geometric design. In our work, we give a geometric criterion, based on understanding the relation between the base loci of the map and its inverse and the syzygies of the entries of the map. We will make special emphasis in the use of exterior algebra methods in the proofs. We will also discuss other known cases of birationality of maps defied over toric varieties.

Hilbert schemes of points of singular curves

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Resumen. The Hilbert scheme of m points on a variety X parametrizes zero-dimensional subschemes of length m in X. Since their introduction by Grothendieck, these spaces have become central not only in algebraic geometry but also in areas like tensor theory and computer vision. A major challenge in studying Hilbert schemes is their intricate geometry, as they satisfy Murphy's Law. For smooth curves, the Hilbert scheme is irreducible and smooth—a property that fails with singular curves. In this joint work with Ángel David Ríos Ortiz, we study Hilbert schemes of points on curves with rational n-fold singularities. We describe their irreducible components, explore their combinatorics and singularities, and show they are reduced.

Distinguishing phylogenetic networks

Marta Casanellas, Jesús Fernández-Sánchez, Elizabeth Gross, Ben Hollering, Seth Sullivant

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Resumen. One of the prior steps to perform inference on phylogenetic networks is ensuring that the phylogenetic network can be identified from the distributions generated by a Markov process on it. Many different authors have provided equations that allow distinguishing a phylogenetic network from another. These equations have been usually obtained by computational algebra software for small networks evolving under simple models such as CFN, JC69, K80 o K81. In this talk we will explain how to obtain equations for networks evolving under equivariant models (which include the previous but also the general Markov model and other submodels) based on rank conditions from flattening matrices. These equations can be used to prove that certain networks are distinguishable.

Maximum likelihood thresholds for colored Gaussian graphical models

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Resumen.Colored Gaussian graphical models are statistical models arising from graphs with a coloring in its vertices and edges. We study maximum likelihood thresholds (MLT) for these models: the minimum number of observations that ensure existence of the maximum likelihood estimator. We extend known results for MLT to the colored setting, determine thresholds for certain families graphs and implement algorithms that exploit the underlying algebraic geometry.

Crossing the Transcendental Divide: From Schottky Groups to Algebraic Curves

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Resumen. Though the uniformization theorem guarantees an equivalence of Riemann surfaces and smooth algebraic curves, moving between analytic and algebraic representations is inherently transcendental. Our analytic curves identify pairs of circles in the complex plane via free groups of Möbius transformations called Schottky groups. We construct a family of non-hyperelliptic surfaces of genus $g \geq 3$ where we know the Riemann surface as well as properties of the canonical embedding, including a nontrivial symmetry group and a real structure with the maximal number of connected components (an M-curve). We then numerically approximate the algebraic curve and Riemann matrices underlying our family of Riemann surfaces.

Referencias

[1] S. Fairchild, Á.D. Ríos Ortiz (2024+). Crossing the transcendental divide: from Schottky groups to algebraic curves. arXiv:2401.10801

Some advances on the Eisenbud-Goto conjecture for projective toric surfaces

Mario González-Sánchez, Ignacio García-Marco, Philippe Gimenez, Mario González-Sánchez

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Resumen.In 1984, Eisenbud and Goto conjectured that the Castelnuovo-Mumford regularity of any reduced and irreducible projective variety was upper bounded by its degree. Over the years, this conjecture was proven true in many cases, e.g. for curves, until 2017, when McCullough and Peeva proved it false in general. However, the conjecture remains open for projective toric varieties. An affirmative answer has been found for smooth toric varieties of any dimension, and many people still believe it is true. In this talk, we will discuss recent progress in proving the conjecture for simplicial toric surfaces using techniques from additive combinatorics.

Involutively stable ideals

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Resumen. We define involutively stable monomial ideals as those ideals whose minimal monomial generating set is at the same time an involutive basis with respect to a certain involutive division. In this talk we study involutively stable ideals with respect to the Pommaret and Janet divisions, and relate them to several important properties of monomial ideals, in particular, componentwise linearity.

Rank-one convexity

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Resumen.Rank-one convexity is a relevant and wild notion in multivariate calculus of variations. We prove that the rank-one convex hull of finitely many 2×2 triangular matrices is a semialgebraic set, defined by linear and quadratic polynomials. We present explicit constructions for five-point configurations and offer evidence suggesting that a similar semialgebraic characterization does not hold in the more general setting of directional convexity.