Loci of Riemann and Klein Surfaces and Related Topics

Equipo organizador

- Milagros Izquierdo (Linköping University)
- Sebastián Reyes Carocca (Universidad de Chile)
- Antonio F. Costa (Universidad Nacional de Educación a Distancia)

Descripción

The special session will be devoted to Riemann and Klein surfaces, and Jacobian varieties and their moduli spaces, with special attention to group actions on surfaces and their Jacobian varieties, automorphisms of surfaces (real and complex), Grothendieck theory of dessins d'enfants (maps and hypermaps) and topological properties of moduli spaces of complex and real curves. The special session will have a computational/combinatorial flavour, with focus on group actions on Riemann surfaces, Klein surfaces and related structures such as abelian varieties or hyperbolic manifolds. The session will consider as well orbifolds in low dimension.
The study of Riemann surfaces with automorphisms constitutes an important meeting point for Group Theory, Geometry and Analysis, and there is considerable current activity in this field. For instance, several recent meetings of the AMS have had special sessions on these and closely related themes, as well as a number of conferences: BIRS, Banff (2017) Canada; the series *Geometry at the Frontier* (2018-2022); within *Congreso Iberoamericano de Geometría* since 2018.

Palabras clave: Compact and non-compact Riemann Surfaces; Klein surfaces; Teichmüller and moduli spaces; Fuchsian and NEC groups; Jacobian Varieties.

Some of the speakers in this session will talk in the proposed session *Teoría de nudos.*

Programa

JUEVES, 22 de enero

11:00 – 11:30	Sebastián Reyes Carocca (Universidad de Chile) $\mathbb{Z}_k^m$-*actions of signature* $(0; k, \overset{n+1}{\ldots}, k)$
11:30 – 12:00	Ewa Kozlowska-Walania (University of Gdansk, Poland) *Extremal configurations of commuting symmetries*
12:00 – 12:30	Angel Carocca (Universidad de La Frontera, Temuco, Chile) *Group Actions on Abelian Varieties*
12:30 – 13:00	Claudia Muñoz (Universidad Autónoma de Madrid) *Uniformization of extremal hyperbolic surfaces with geodesic boundary and/or punctures*
13:00 – 13:30	José Javier Etayo (Universidad Complutense de Madrid) *No hay ningún grupo de género real 72*
15:30 – 16:00	Antonio Breda (Universidad de Aveiro) *On Singular Map-Spectra Groups*
16:00 – 16:30	Adrián Bacelo (Universidad Complutense de Madrid) *Comparing the real genus and the symmetric crosscap number of a group*
16:30 – 17:00	Pietro Speziali (Universidade Estadual de Campinas) *The hunt for compact Riemann Surfaces with a transitive action on Weierstrass points*
17:00 – 17:30	Federico Zerbini (UNED) *Polylogarithms on Riemann surfaces*
18:00 – 18:30	Elba García-Failde (Universidad Politécnica de Cataluña) *Volumes of moduli spaces of bordered Klein surfaces*
18:30 – 19:00	Rubí Rodríguez (Universidad de La Frontera, Temuco, Chile) *Gluing Riemann surfaces and their Jacobians*
19:00 – 19:30	Raquel Díaz (Universidad Complutense de Madrid) *Boundary of equisymmetric strata of Riemann surfaces with abelian automorphism groups*
19:30 – 20:00	Ewa Tyszkowska (University of Gdansk, Poland) *Clifford bundles over Klein surfaces*

VIERNES, 23 de enero

11:00 – 11:30	Rubén Hidalgo (Universidad de La Frontera) *On quasiconformal equivalence of Schottky regions*
11:30 – 12:00	Anita Rojas (Universidad de Chile) *On Non-Normal Subvarieties of the Moduli Space of Riemann Surfaces*
12:00 – 12:30	Enrique Artal (Universidad de Zaragoza) *Plane octic curves with* $6\,\mathbb{E}_6$ *singular points and non hyperelliptic Riemann surfaces of genus* 3
12:30-13:00	Ernesto Girondo (Universidad Autónoma de Madrid) *The minima of the geodesic length function for uniform filling curves*

$\mathbb{Z}_k^m$-actions of signature $(0; k, \overset{n+1}{\cdots}, k)$

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Resumen. In this talk we consider $\mathbb{Z}_k^m$-actions of signature $(0; k \overset{n+1}{\cdots}, k)$, namely, pairs (S, N) where S is a compact Riemann surface endowed with a group of automorphisms $N \cong \mathbb{Z}_k^m$ acting on S with signature $(0; k, \overset{n+1}{\cdots}, k)$. We shall discuss some recent results concerning the topological classificaction of such actions, particularly in the case they admit extra automorphisms. This is a joint work with Rubén A. Hildalgo (Universidad de La Frontera, Chile).

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Extremal configurations of commuting symmetries

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Resumen. We consider configurations of commuting symmetries that admit the maximal possible total number of ovals. We show all the possible topological symmetry types of underlying surfaces and provide their defining real equations. This is a joint work with Peter Turbek.

Group Actions on Abelian Varieties

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Resumen. Let $\mathcal{A}$ be an abelian variety. It is known that there exists a bijection between the set of abelian subvarieties of $\mathcal{A}$ and the set of symmetric idempotents in $\mathrm{End}_{\mathbb{Q}}(A)$. When considering a finite group G acting on an abelian variety $\mathcal{A}$, determining idempotent elements $f \in \mathbb{Q}[G]$, especially those associated with subgroups, and studying the geometric properties and the endomorphism ring of the subvariety $\mathrm{Image}(f)$, have become central problems in this field of study.

Let us consider subgroups $K \trianglelefteq H \leq G$ such that (K, H) forms a Shoda pair. Define

1. $$\epsilon(K,H) = \begin{cases} p_K & \text{if} \quad K = H \\ \displaystyle\prod_{M \in \mathcal{M}(H/K)} (p_K - p_M) & \text{if} \quad K \neq H \end{cases}$$

   where $\mathcal{M}(H/K)$ denotes the set of minimal normal subgroups of H containing K properly and
   $$p_K = \frac{1}{|K|} \sum_{k \in K} k.$$

2. $$e(G,K,H) = \sum_{g \in T} \epsilon(K,H)^g$$

   where T is a right transversal of $\mathbf{C}_G(\epsilon(K,H))$ in G.

In this communication, we present some results concerning the geometric description of $\mathrm{Image}(\epsilon(K,H))$ and $\mathrm{Image}(P_K - P_M)$, for each $M \in \mathcal{M}(H/K)$.

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Uniformization of extremal hyperbolic surfaces with geodesic boundary and/or punctures.

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Resumen. An important geometric feature of hyperbolic surfaces, ie. the topological surfaces whose universal covering space is the hyperbolic plane, is the largest radius of a metric disc properly embedded in a given surface, known as its *injectivity radius*. In 1996, C. Bavard [1] showed that the maximal injectivity radius of any compact orientable hyperbolic surface without boundary is sharply bounded above by a certain *extremal radius* that depends only on the topology of the surface. This upper bound was later generalized for non-orientable closed hyperbolic surfaces and, in the following years, several authors studied the closed hyperbolic surfaces that contain an embedded metric disc of that extremal radius, the so-called *extremal surfaces*, by using the uniformization of hyperbolic surfaces by NEC groups as in [4].

However, in the last decade, a different approach has been proposed by J. deBlois and K. Romanelli along the papers [2] and [3]. This new setting has been essential in determining the dependence of the extremal radius of general hyperbolic surfaces of finite type, which may have punctures and/or geodesic boundary, with respect to the underlying topology of the surface. In this talk, based on a recent joint work [5] with Ernesto Girondo, we translate all these results back into the language of the uniformization by NEC groups and we apply this different point of view for solving some interesting problems about non-closed extremal surfaces, such as counting the total number of extremal surfaces given their topological data or studying conditions on the existence of multiple extremal discs within a given surface.

Referencias

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No hay ningún grupo de género real 72

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Resumen. Todo grupo finito G actúa como grupo de automorfismos de diversas superficies de Klein con borde. Al menor de los géneros algebraicos de estas superficies se le llama género real $\rho(G)$ del grupo G. Se sabe que todos los enteros positivos impares son género real de algún grupo. En cambio, no todos los pares lo son. C. L. May recopiló una serie de familias de grupos, a partir de las cuales obtuvo sucesiones aritméticas de géneros reales pares con las que se cubre gran parte del conjunto de los números pares. En particular, resultó que los números 2, 12 y 24 no son género real de ningún grupo. May se planteó si este era un fenómeno propio de números pequeños, o bien existen otros huecos en el espectro del género real, esto es, números N que no son el género real de ningún grupo. El primer número para el que esta cuestión permanecía abierta es 72. En el presente trabajo probamos que no existe ningún grupo de género real 72.

Abstract. Every finite group G acts as an automorphism group of several bordered Klein surfaces. The minimal genus of these surfaces is called the real genus $\rho(G)$ of the group G. It is known that all odd positive integers are the real genus of some group. On the contrary, not all even integers are. C. L. May compiled a series of families of groups, from which he obtained arithmetic sequences of even numbers which are real genus of some group, covering a large part of the even numbers. In particular, it results that 2, 12 and 24 are not the real genus of any group. May asked on whether this is as a question of small numbers, or else there are other gaps in the spectrum of the real genus, that is to say, numbers N such that there are no groups of real genus N. The first number on which the question remained unsolved is 72. In the present work we prove that there is no group of real genus 72.

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On Singular Map-Spectra Groups

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Resumen. In a recent paper [1] we encountered certain peculiarities in the Euler characteristics of surfaces determined by map-generating pairs (a, z) in some groups G. Usually, Euler characteristics differ depending on the restricted regular maps that the pair represents. We explores one such peculiarity in the context of singular map-spectra groups. Given a group G generated by an involution a and an element z of order n, each map-generating pair (a, z) in G corresponds to three distinct restricted regular maps: a regular oriented map, a regular bi-oriented map, and a regular face-bi-oriented map, each determining its own Euler characteristic. A singular map-spectra group is defined as a group for which every map-generating pair (a, z) yields the same Euler characteristic, regardless of whether it represents a regular oriented map, a regular bi-oriented map, or a regular face-bi-oriented map.

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Comparing the real genus and the symmetric crosscap number of a group

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Resumen. Given a finite group G, there exist Klein surfaces, bordered X and unbordered non-orientable S, such that G acts as an automorphism group of X and of S. The minimum algebraic genus $\rho(G)$ of the surfaces X is called the real genus of G, and the minimal topological genus $\tilde{\sigma}(G)$ of the surfaces S is the symmetric crosscap number of G. In this work we study the relation between the real genus and the symmetric crosscap number of a group G and how both parameters can be compared. For instance, we see that there exist groups G such that the difference $\tilde{\sigma}(G) - \rho(G) = t$ for all even negative numbers t. In order to get it, we correct some inaccuracies in previous works, on these parameters for the groups $C_m \times D_n$ and $D_m \times D_n$. On the other hand, for some important families of groups, we prove that $\tilde{\sigma}(G)=\rho(G)+1$. We use it to eliminate possible gaps in the symmetric crosscap spectrum, enforcing the conjecture that 3 is in fact the unique gap.

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The hunt for compact Riemann Surfaces with a transitive action on Weierstrass points

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Resumen. Let X be a compact Riemann surface of genus $g > 2$. By the celebrated Hurwitz bound, the automorphism group $\mathrm{Aut}(X)$ is finite and has size at most $84(g-1)$. A classical result due to Lewittes states that if an automorphism σ of X fixes more than four points, then all of these fixed points are Weierstrass points. Moreover, $\mathrm{Aut}(X)$ acts on the set W of Weierstrass points of X. A natural question is whether this action can be transitive.

This phenomenon is very rare: only a handful of examples are known, and until recently, the only infinite family was provided by the famous hyperelliptic Accola–Maclachlan curves, which exist in every genus. In our recent work [1], we provided the first infinite family of pairwise non-isomorphic, non-hyperelliptic curves with a transitive action on W; all of these surfaces have genus 3.

On the other hand, there are genera where no such non-hyperelliptic surfaces exist; the smallest case is genus 6. Moreover, as far as the authors know, no non-hyperelliptic examples are known for $g > 7$. In this talk, we will present our ongoing research on this problem, with particular emphasis on identifying new examples in low genus.

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Polylogarithms on Riemann surfaces

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Resumen. Polylogarithms are classical special functions that appear in several different domains, which include number theory, hyperbolic geometry, algebraic K-theory, deformation quantization and particle physics. They encode geometric information about (configuration spaces of) genus-zero Riemann surfaces, as they are related with their prounipotent fundamental group. We will report on a recent general construction of polylogarithms on Riemann surfaces, obtained in a series of joint works with B. Enriquez, and motivated by their appearance in recent computations from particle physics.

Volumes of moduli spaces of bordered Klein surfaces

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Resumen. In 2006, Mirzakhani produced a recursion to effectively compute the Weil–Petersson volumes of moduli spaces of genus g hyperbolic surfaces with n marked geodesic boundaries. I will present a generalisation of her recursion that allows to compute volumes of moduli spaces of bordered Klein surfaces. Combining Mirzakhani's recursion with ours, produces a recursion for total volumes of orientable and non-orientable hyperbolic surfaces. When the surface is non-orientable, the volume is considered with respect to a top-form introduced by Norbury in 2008. However, these volumes diverge when the lengths of 1-sided geodesics approach 0; in 2017, Gendulphe proposed to consider regularised volumes whose systole of 1-sided geodesics is greater than epsilon, for epsilon small enough. Making use of a generalisation of McShane identity due to Norbury, we are able to obtain a simple, exact expression for the volume of the moduli space of Klein bottles and a recursion for generic topologies that fully capture the dependence on the geometric regularisation parameter epsilon. I will finish by briefly commenting on the relation to a refinement of the universal procedure of topological recursion.

Agradecimientos. This talk will be based on joint work with P. Gregori and K. Osuga.

Gluing Riemann surfaces and their Jacobians

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Resumen. En esta charla discutiremos el “pegado de curvas” y de sus Jacobianos más allá de los casos usuales de géneros uno y dos.

Este problema tiene aplicaciones aritméticas y algebraicas interesantes, muchas aún por descubrir.

In this talk we will discuss the “gluing of curves” and their Jacobians beyond the usual cases of genera one and two.

This problem has arithmetic and algebraic interesting applications, many of them still to be discovered.

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Boundary of equisymmetric strata of Riemann surfaces with abelian automorphism groups

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Resumen. The moduli space M_g of Riemann surfaces of genus g is an orbifold obtained as the quotient of Teichmüller space under the mapping class group. Its branch locus is naturally stratified, each strata corresponding to those Riemann surfaces whose automorphic groups act in the same (topological) way. The Deligne-Mumford compactification of M_g consists on adding the so called stable Riemann surfaces. A natural question is which topological types of stable Riemann surfaces appear in the boundary of a given equisymmetric stratum. A general procedure to determine this boundary is given in a previous work by the authors [1]. In this talk we particularize this procedure to strata with abelian actions, giving a quite complete answer for the cases of p-adic and hyperelliptic actions.

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Clifford bundles over Klein surfaces

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Resumen. Let V be a real vector space and let $Q : V \to \mathbb{R}$ be a quadratic form with the signature (r, s). The *Clifford algebra* is an associative algebra $\mathrm{Cl}_{r,s}$ together with a linear map $i : V \to \mathrm{Cl}_{r,s}$ such that $i(v)^2 = -Q(v)1$ for all $v \in V$. Moreover, for any associative algebra C with unit 1_C and a linear map $F : V \to C$ satisfying $F(v)^2 = -Q(v)1_C$ for all $v \in V$, there exists a unique algebra homomorphism $\tilde{F} : \mathrm{Cl}_{r,s} \to C$ such that $\tilde{F} \circ i = F$. In particular, by taking $C = \mathrm{Cl}_{r,s}$ and $F = i \circ f$ for an ortogonal map f induced by $f(v) = -v$ for all $v \in V$, we get an automorphism $\tilde{F}$ of $\mathrm{Cl}_{r,s}$. The subalgebra of $\mathrm{Cl}_{r,s}$ fixed by this automorphism is denoted by $\mathrm{Cl}^+_{r,s}$.

The *Pin group* is a multiplicative subgroup $\mathrm{Pin}(r, s)$ of $\mathrm{Cl}_{r,s}$ generated by the set $\{i(v) : v \in V \ \wedge Q(v) = \pm 1\}$ and $\mathrm{Pin}(r, s) \bigcap \mathrm{Cl}^+_{r,s}$ is called the *Spin group*. A subgroup $M_{r,s}$ of $\mathrm{Pin}(r, s)$ generated by the images of an ortonormal basis of V is called the *base group*.

Let Y be a Klein surface Y of algebraic genus $d > 1$ with q boundary components and let $p = d+1-q$. We construct a principal $\mathrm{Pin}(p, q)$-bundle over Y and a principal $\mathrm{Spin}(p, q)$-bundle over the canonical double cover Y^+ of Y. We study vector bundles over Y and Y^+ associated to representations of the Pin and Spin groups and we show how isomorphisms of Clifford algebras affect these vector bundles. We represent a Klein surface as the connected sum of two other Klein surfaces and we find the relationships among their vector bundles.

On quasiconformal equivalence of Schottky regions

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Resumen. This talk concerns the general question: When are two homeomorphic Riemann surfaces quasiconformally equivalent?
In the case of finite-type Riemann surfaces the answer is well known; it depends on the genus, the number of punctures, and the number of hyperbolic boundaries.
On the other hand, for infinite-type Riemann surfaces, this is rather complicated. For instance, it is well known that the homology cover of a closed Riemann surface of genus at least two is homeomorphic to the Loch Ness monster (the unique, up to homeomorphisms, orientable surface of infinite genus and with exactly one end). In this setting, it is known the following (a Fuchsian type of Torelli's Theorem): two hyperbolic closed Riemann surfaces are biholomorphically equivalent if and only if their homology covers are. Of course, two surfaces of the same genus have quasiconformally equivalent homology covers. The converse is an open problem.
In [1], Shiga proved that, if G_1 and G_2 are Schottky groups of (possible different) rank at least two, then the complements of their (Cantor) limit sets are quasiconformally equivalent. Generalizing this result, we provide a general solution for those Riemann surfaces $S = \widehat{\mathbb{C}} \setminus \Lambda$, where Λ is a Cantor set being the limit set of a finitely generated Kleinian group. As a consequence, we obtain that there are exactly four different Teichmüller spaces of Cantor limit sets of finitely generated Kleinian groups.

Referencias

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On Non-Normal Subvarieties of the Moduli Space of Riemann Surfaces

En honor a Gabino González-Diez, por su extraordinaria contribución a la Matemática y su gran calidad humana.

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Resumen.
We consider the irreducible subvarieties of the moduli space $\mathcal{M}_g$ of compact Riemann surfaces of genus $g \geq 2$ denoted by $\mathcal{M}_g(H, s, \theta)$. They represent isomorphism classes of surfaces endowed with the action of a group H with topological class determined by the surface-kernel epimorphism $\theta : \Delta \to H$, where Δ is a Fuchsian group of signature s. Such subvarieties are contained in the singular locus of $\mathcal{M}_g$ and, in general, are non-smooth. In [5], we consider certain subvarieties of this type and address the problem of determining which among them are non-normal.
We use a key fact proved by Gabino González-Diez and William Harvey [3] which states that the existence of a non-normal point $[X_0] \in \mathcal{M}_g(H, s, \theta)$ is closely related to the existence of groups of automorphisms of X_0 isomorphic to H acting on X_0 with topological class determined by θ and that are non-conjugate in the full automorphism group of X_0.
Gabino González-Diez and Rubén Hidalgo constructed in [4] the first example of a non-normal subvariety of this type associated with regular cyclic covers of the projective line. Later, Mariela Carvacho in [1] generalised this construction. Javier Cirre in [2] constructed non-normal subvarieties of this type associated to regular non-cyclic covers of $\mathbb{P}^1$.
In this talk, we will explain these results and our work [5], where we construct subvarieties of this type in several moduli spaces. In particular, we show that for each $g \geq 2$ the moduli space $\mathcal{M}_g$ contains a non-normal subvariety of type $\mathcal{M}_g(H, s, \theta)$ where s is a genus zero signature.

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Plane octic curves with $6\,\mathbb{E}_6$ singular points and non hyperelliptic Riemann surfaces of genus 3

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Resumen. In [1], several non isotopic families of plane curves of degree 8 having exactly 6 singular points (of type $\mathbb{E}_6$) are given, on both the algebraic and the symplectic categories. Unfortunately a complete classification was not obtained. In this talk, the goal is to share some approaches to this classification which relate this problem with some special divisors in non-hyperelliptic Riemann surfaces of genus 3.

Referencias

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The minima of the geodesic length function for uniform filling curves

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Resumen. Un resultado famoso, demostrado en primer lugar por Kerckhoff, establece que la función de longitud geodésica determinada por una curva filling tiene un mínimo en el espacio de Teichmüller, y además que este mínimo es único. En la charla presentaremos explícitamente estos mínimos para una cierta clase de curvas, mostrando que se alcanzan en las superficies de Grothendieck-Belyi determinadas por ciertos dessins d'enfants que se asocian de manera natural con las curvas filling de partida.

Abstract. A famous result, first proved by Kerckhoff, states that the geodesic length function determined by a curve that fills up a compact surface has a minimum in Teichmüller space, and that this minimum is unique. We exhibit explicitly these minima for a certain class of curves, showing that they are attained at the Grothendieck–Belyi surfaces determined by some dessin d'enfants naturally associated to these filling curves.

Referencias

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