Teoría de números

Equipo organizador

- Óscar Rivero (Universidade de Santiago de Compostela)
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Descripción

La teoría de números es un área que goza de gran tradición en España. Son varios los polos geográficos donde se concentran investigadores de esta rama (Barcelona, Lleida, Madrid, Málaga, Sevilla, Zaragoza o Santiago) y la diversidad de temas tratados cubre un amplio espectro dentro de la disciplina: geometría aritmética, métodos p-ádicos, teoría analítica, teoría de Hopf–Galois o métodos computacionales. Pretendemos que esta sesión sirva para poner en contacto a distintos investigadores, crear sinergias y potenciar colaboraciones dentro de la comunidad de teoría de números en España.

Palabras clave: Teoría de números; Geometría aritmética; Representaciones de Galois; Funciones L; Teoría de Hopf–Galois.

Programa

LUNES, 19 de enero

15:30 – 16:00	José María Tornero (Universidad de Sevilla) *Cuerpos de Heegner*
16:00 – 16:30	Beatriz Barbero (University College Dublin) *On the existence of morphisms between certain Artin–Schreier curves*
16:30 – 17:00	Daniel Martínez Marqués (Universidad Autónoma de Madrid) *The refined class number formula for Drinfeld modules*
17:00 – 17:30	Ariel Pacetti (Universidade de Aveiro) *Motivos hipergeométricos y la ecuación de Fermat generalizada*

MARTES, 20 de enero

11:00 – 11:30	Anna Río (Universitat Politècnica de Catalunya) *Extensions of braces*
11:30 – 12:00	Montse Vela (Universitat Politècnica de Catalunya) *Estructuras Hopf-Galois de los puntos de tres y seis torsión de la lemniscata*
12:00 – 12:30	Daniel Gil-Muñoz (Charles University of Prague) *Estructura aditiva de cuerpos cúbicos más simples de índice primo*

15:30 – 16:00	Armando Gutiérrez Terradillos (Aarhus University) *On periods and L-functions for* $\mathrm{GU}(2,2) \times \mathrm{GL}(2)$
16:00 – 16:30	Óscar Rivero (Universidade de Santiago de Compostela) *An anticyclotomic Euler system for Hilbert cuspforms over a real quadratic field*
16:30 – 17:00	Carlos de Vera Piquero (Universidad de Zaragoza) *On p-adic L-functions for* $\mathrm{GL}_2 \times \mathrm{GL}_3$ *and Artin formalism*
17:00 – 17:30	Martí Roset (Sorbonne Université) *Rigid cocycles for* SL_n *and their values at special points*

18:00 – 18:30	Mar Curcó-Iranzo (Utrecht University) *Generalised Jacobians and modular curves*
18:30 – 19:00	Ignacio Muñoz Jiménez (Università degli Studi di Genova) *Quaternionic big Heegner points over totally real fields*

Cuerpos de Heegner

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Resumen. Este trabajo está aún en desarrollo y su autoría es compartida con Joan Carles Lario y Miguel Pineda. La idea consiste en estudiar cuerpos cúbicos que generalizan los usados por Heegner para resolver el problema de los cuerpos cuadráticos imaginarios de clase 1. Por el camino aparecerán curvas de género 1 (evidentemente), problemas de ramificación (cómo no) y una cantidad considerable de cálculos (spoiler alert).

On the existence of morphisms between certain Artin–Schreier curves

Beatriz Barbero, Gary McGuire, Stefano Lia

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Resumen. It is well known that given the two Artin–Schreier curves of the form $B_m\colon y^p - y = x^m$ and $B_d\colon y^p - y = x^d$, defined over $\mathbb{F}_p$, if $d \mid m$ then there exists a surjective morphism $B_m \to B_d$. In this work we are interested in studying when the converse of this statement is true. In particular, we consider the case when $m = p^k + 1$ and $d = p^r + 1$. This is a joint work with Gary McGuire and Stefano Lia.

The refined class number formula for Drinfeld modules

Daniel Martínez Marqués, María Inés de Frutos Fernández, Daniel Macías Castillo

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Resumen. In 2012, Taelman proved an analogue of the analytic class number formula for Drinfeld modules. In joint work with María Inés de Frutos Fernández and Daniel Macías Castillo, we state and prove an equivariant refinement of this formula, relating the special L-value defined in a Whitehead group with the refined Euler characteristic of a complex constructed from the Drinfeld exponential. In this talk, we will review Taelman's work and discuss the formulation of the refined formula, commenting on how we circumvent the difficulties arising in noncommutative settings.

Motivos hipergeométricos y la ecuación de Fermat generalizada

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Resumen. El objetivo de esta charla es dar una introducción a los motivos hipergeométricos así cómo enunciar sus propiedades más importantes. Aprovecharemos para mostrar cómo el uso de motivos hipergeométricos se encuadra bien en el programa de Darmon para el estudio de la ecuación de Fermat generalizada $x^p + y^q = z^r$.

Extensions of braces

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Resumen. In group theory, semidirect products provide an *external* construction for split extensions. In the theory of skew braces, however, a general notion of semidirect product is still lacking, despite the existence of several partial approaches. When considering split extensions separately for the additive and multiplicative structures, certain compatibility conditions arise. Our aim is to formulate the problem in (skew brace-)cohomological terms, with the ultimate goal of establishing a coherent definition for split extensions in the context of skew braces.

Estructuras Hopf-Galois de los puntos de tres y seis torsión de la lemniscata

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Resumen. Estudiamos el cuerpo de números generado por las coordenadas de los puntos de tres (o seis) torsión de la lemniscata. Esta extensión no es de Galois pero admite seis estructuras Hopf-Galois diferentes. Daremos la descripción de estas estructuras calculando el álgebra de Hopf asociada a una de ellas y analizando y calculando el retículo de subcuerpos de esta extensión

Estructura aditiva de cuerpos cúbicos más simples de índice primo

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Resumen. La familia de los cuerpos cúbicos más simples (simplest cubic fields) es una familia paramétrica de cuerpos numéricos cúbicos introducidos por Shanks en 1974, que poseen propiedades aritméticas que los hacen un objeto de estudio bastante accesible. En esta charla nos restringiremos a aquellos cuerpos cúbicos más simples cuyo elemento generador tiene índice primo, y trataremos el problema de la estructura aditiva del anillo de enteros de un cuerpo cúbico más simple y, en particular, de sus elementos algebraicos totalmente positivos que son aditivamente indescomponibles, en el sentido de que no se pueden descomponer como suma de otros elementos algebraicos totalmente positivos.

On periods and L-functions for $\mathrm{GU}(2,2) \times \mathrm{GL}(2)$

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Resumen. The study of periods of automorphic forms is a key theme in the Langlands program and has become central to understanding the structure and properties of automorphic representations. In recent years, the conjectures of Sakellaridis and Venkatesh (and then Ben-Zvi, Sakellaridis, and Venkatesh) in the context of spherical varieties has led to a deeper understanding of automorphic periods and their relation to special values of L-functions. In this talk, I present work in progress aimed at looking at certain non-spherical cases. Precisely, I will describe a new integral representation of the degree 12 "exterior square x standard" L-functions on generic cusp forms on $\mathrm{GU}(2,2) \times \mathrm{GL}(2)$ (or $\mathrm{GL}(4) \times \mathrm{GL}(2)$) and how it can be used to relate the non-vanishing of its central value to a certain cohomological and non-spherical period.

An anticyclotomic Euler system for Hilbert cuspforms over a real quadratic field

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Resumen. Let g be a Hilbert cuspform of parallel weight over a real quadratic field F and let $\mathrm{As}(V_g)$ denote the Asai representation associated with g. Let χ be a Hecke character of an imaginary quadratic field K for which the G_K-representation $\mathrm{As}(V_g)(\chi)$ is conjugate self-dual. In this talk we will explain how to construct an Euler system for $\mathrm{As}(V_g)(\chi)$. Expected applications include results towards the Bloch–Kato conjecture and towards an Iwasawa Main Conjecture for this representation.

On p-adic L-functions for $\mathrm{GL}_2 \times \mathrm{GL}_3$ and Artin formalism

Carlos de Vera Piquero, Aprameyo Pal, Kazim Büyükboduk, Daniele Casazza

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Resumen. I will report on an explicit construction of p-adic L-functions for $\mathrm{GL}_2 \times \mathrm{GL}_3$ attached to Hida families of modular forms f and g, via p-adic interpolation of the so-called Shintani and Saito–Kurokawa lifts. I will also explain how these p-adic L-functions conjecturally arise as factors of certain triple product p-adic L-functions, yielding a p-adic Artin formalism for them, and show evidence towards this. The talk will be based on a series of works in collaboration with A. Pal, D. Casazza, and K. Büyükboduk.

Rigid cocycles for SL_n and their values at special points

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Resumen. The theory of complex multiplication implies that the values of modular functions at CM points belong to abelian extensions of imaginary quadratic fields. In this talk, we propose a conjectural approach to extending this phenomenon to the setting of totally real fields. Generalizing the work of Darmon, Pozzi, and Vonk, we construct rigid cocycles for SL_n, which play the role of modular functions, and define their values at points associated with totally real fields. The construction of these cocycles originates from a topological source: the Eisenstein class of a torus bundle.

Generalised Jacobians and modular curves

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Resumen. The Jacobian $J_0(N)$ of the modular curve $X_0(N)$ has received much attention within arithmetic geometry for its relation with cusp forms and elliptic curves. The group of $\mathbb{Q}$-rational torsion points on $J_0(N)$ controls the unramified cyclic covers of $X_0(N)$. A conjecture of Ogg predicted that, for N prime, the torsion of this group comes all from the cusps and is related to the so called Shimura covering - the maximal unramified abelian cover of $X_0(N)$. The statement was proved by Mazur and later generalised to arbitrary level N into what we call generalised Ogg's conjecture.
Consider now the generalised Jacobian $J_0(N)_{\mathbf{m}}$ with respect to a cuspidal modulus $\mathbf{m}$, i.e., a rational effective divisor with support at the cusps. This algebraic group is also be related to the arithmetic of $X_0(N)$ through the theory of modular forms. Now the $\mathbb{Q}$-rational torsion of $J_0(N)_{\mathbf{m}}$ is related to cyclic covers of $X_0(N)$ ramified at the cusps. In the talk we will introduce generalised Jacobians, we will present results that compute the $\mathbb{Q}$-rational torsion of $J_0(N)$ for N an odd integer and we will discuss some ongoing research on maximal abelian coverings of $X_0(p)$ unramified outside cusps.

Quaternionic big Heegner points over totally real fields

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Resumen. The seminal work of Kolyvagin and Gross–Zagier in the late 1980s, centered on special points on elliptic curves known as Heegner points, remains one of the most significant advances toward the Birch and Swinnerton-Dyer conjecture. Since then, their methods have been extended to many contexts, with modular forms playing a central role. In this talk, I will review Howard's construction of "big Heegner points", which generalize classical Heegner points to p-adic families of modular forms. Howard's framework makes it possible to deform the classical setting and thereby study intricate or previously inaccessible cases. I will then outline ongoing work extending these ideas to Hilbert modular forms, yielding a totally real counterpart.