

Optimal regularity of radial solutions to the general Hardy-Hénon equation on the ball.

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Resumen. This contribution is devoted to the study of semi-stable radial solutions $u \in H^1(B_1)$ of $-\Delta u = |x|^\alpha f(u)$ in $B_1 \setminus \{0\}$, where $f \in C^1(\mathbb{R})$ is a general nonlinearity, $\alpha > -2$ and B_1 is the unit ball of $\mathbb{R}^N$, $N > 1$. We establish the boundedness of such solutions for dimensions $2 \leq N < 10 + 4\alpha$ and sharp pointwise estimates in the case $N \geq 10 + 4\alpha$. In addition, we provide, for this range of dimensions, a large family of semi-stable radially decreasing unbounded $H^1(B_1)$ solutions.

Palabras clave: Hardy-Henon equation, regularity estimates, semi-stable solutions.

Referencias

- [1] H. Brezis, J. L. Vázquez (1997). Blow-up solutions of some nonlinear elliptic problems. *Rev. Mat. Univ. Complut. Madrid*, 10, no. 2, 443–469.
- [2] L. A. Caffarelli, B. Gidas, J. Spruck (1989). Asymptotic symmetry and local behavior of semilinear elliptic equations with critical Sobolev growth. *Comm. Pure Appl. Math.*, 42, no. 3, 271–297.
- [3] S. Chandrasekhar (1957). *An Introduction to the Study of Stellar Structure*. Dover Publications, New York.
- [4] W. Dai, S. Peng (2024). Liouville theorems of solutions to mixed order Hénon-Hardy type system with exponential nonlinearity. *Adv. Nonlinear Stud.*, 24, no. 2, 335–358.
- [5] E. N. Dancer, Y. Du, Z. Guo (2011). Finite Morse index solutions of an elliptic equation with supercritical exponent. *J. Differential Equations*, 250, no. 8, 3281–3310.
- [6] J. Dávila, L. Dupaigne, A. Farina (2011). Partial regularity of finite Morse index solutions to the Lane-Emden equation. *J. Funct. Anal.*, 261, no. 1, 218–232.
- [7] A. T. Duong (2024). Liouville type theorems for fractional elliptic systems with coupled terms. *Math. Inequal. Appl.*, 27, no. 2, 261–272.
- [8] L. Dupaigne, J. Dávila, J. Wei (2017). On the fractional Lane-Emden equation. *Trans. Amer. Math. Soc.*, 369, no. 9, 8377–8424.
- [9] A. Farina (2007). On the classification of solutions of the Lane-Emden equation on unbounded domains of $\mathbb{R}^N$. *J. Math. Pures Appl.*, 87, no. 5, 537–561.
- [10] M. Fazly, Y. Hu, W. Yang (2021). On stable and finite Morse index solutions of the nonlocal Hénon–Gelfand–Liouville equation. *Calc. Var. Partial Differential Equations*, 60, no. 11.

- [11] B. Gidas, J. Spruck (1981). A priori bounds for positive solutions of nonlinear elliptic equations. *Comm. Partial Differential Equations*, 6, no. 8, 883–901.
- [12] S. Hasegawa, N. Ikoma, T. Kawakami (2021). On weak solutions to a fractional Hardy–Hénon equation: Part I: Nonexistence. *Commun. Pure Appl. Anal.*, 20, no. 4, 1559–1600.
- [13] S. Hasegawa, N. Ikoma, T. Kawakami (2023). On weak solutions to a fractional Hardy–Hénon equation, Part II: Existence. *Nonlinear Anal.*, 227, 113–165.
- [14] A. Hyder, W. Yang (2021). Partial regularity of stable solutions to the fractional Gel’fand–Liouville equation. *Adv. Nonlinear Anal.*, 10, no. 1, 1316–1327.
- [15] S. Kim, Y. Lee (2022). Finite Morse Index Solutions of the Fractional Hénon–Lane–Emden Equation with Hardy Potential. *Taiwanese J. Math.*, 26, no. 2, 251–283.
- [16] P. Poláčik, P. Quittner, P. Souplet (2007). Singularity and decay estimates in superlinear problems via Liouville-type theorems. I. Elliptic equations and systems. *Duke Math. J.*, 139, no. 3, 555–579.
- [17] S. Villegas (2012). Sharp estimates for semi-stable radial solutions of semilinear elliptic problems. *J. Funct. Anal.*, 262, 3394–3408.
- [18] C. Wang, D. Ye (2012). Some Liouville theorems for Hénon type elliptic equations. *J. Funct. Anal.*, 262, no. 4, 1705–1727.

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