Iterative PageRank and fixed points

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Resumen. Let us consider a directed graph $\zeta = (V, E)$ with $V = \{1, 2, \ldots, N\}$ the set of nodes and E the set of edges and let denote by $\mathbf{e} = (1, \ldots, 1)^T \in \mathbb{R}^{N\times 1}$, where the superscript T indicates the matrix transposition. If P_A denotes the row-normalization of the adjacency matrix A of the graph ζ, in the absence of dangling nodes, the Google matrix is defined by $G = \lambda P_A + (1-\lambda)\mathbf{e}\mathbf{v}^T$, where $\lambda \in [0, 1]$ is the damping factor and $\mathbf{v} \in \mathbb{R}^{N\times 1}$ satisfying $\mathbf{v} > 0$ and $\mathbf{v}^T\mathbf{e} = 1$ is the personalization vector.

The PageRank vector $\boldsymbol{\pi} \in \mathbb{R}^{N\times 1}$ is the unique positive eigenvector of G^T associated to the eigenvalue 1, that is, $\boldsymbol{\pi} > 0$, $\boldsymbol{\pi}^T\mathbf{e} = 1$ and $G^T\boldsymbol{\pi} = \boldsymbol{\pi}$. In fact, it is well-known [1] that the PageRank vector can be expressed in the closed-form $\boldsymbol{\pi}^T = (1-\lambda)\mathbf{v}^T(I - \lambda P_A)^{-1} = \mathbf{v}^T X(\lambda)$, where $X(\lambda) = (1-\lambda)(I - \lambda P_A)^{-1}$. At this point, what if we compute a new PageRank vector using $\boldsymbol{\pi} \in \mathbb{R}^{N\times 1}$ as a new personalization vector? Is there a limit for the iteration of the PageRank vector under the matrix $X(\lambda)$?

In this talk we study the convergence of the PageRank vector under the iteration of the matrix $X(\lambda)$ when the row-normalization matrix P_A is irreducible, that is, when there is no permutation matrix R of nodes V such that $R^T P_A R$ is upper-triangular. Additionally, we investigate the convergence of the Power Iteration Method applied to the matrix $X(\lambda)$ for the case when the row-normalization matrix P_A is no longer irreducible, that is, there exists a permutation matrix R such that the matrix $R^T P_A R$ has the form,

$$R^T P_A R = \begin{pmatrix} P_{A_1} & B \\ 0 & P_{A_2} \end{pmatrix}$$

where P_{A_1} and P_{A_2} are both square non-negative and irreducible matrices of order n_1 and n_2, respectively, $n_1 + n_2 = N$, and B is a non-negative matrix of order $n_1 \times n_2$. We show that the recursive sequence $\{\mathbf{x}_k\}_{k>1}$ defined by $\mathbf{x}_{k+1}^T = \mathbf{x}_k^T X(\lambda) = \mathbf{v}^T X(\lambda)^k$, $(k > 1)$ converges to a non-zero vector which, surprisingly, is related to the Perron vectors of matrices P_{A_1} and P_{A_2} [2].

Palabras clave: PageRank vector; irreducible matrix; Perron-Frobenius theory; Power Iteration Method.

Referencias

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[2] C.D., Meyer, Matrix Analysis and Applied Linear Algebra, Society for Industrial and Applied Mathematics, Philadelphia, 2000.

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