Numerical approximation of singularly perturbed convection-diffusion elliptic problems with a non-smooth forcing term

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Resumen. Singularly perturbed elliptic problems of convection–diffusion type with smooth coefficients have been extensively analyzed in the literature (see, for example, [2]). The diffusion coefficient (or singular perturbation parameter) can take arbitrarily small values, leading to convection-dominated behavior. The solution of such problems exhibits a multiscale character, with large gradients confined to narrow regions near the boundary and at the corners of the domain, known respectively as boundary and corner layers. In this work, the forcing term is additionally assumed to be non-smooth, either containing an interior layer or being discontinuous across an interface orthogonal to the flow. The former can be viewed as a regularization of the latter, and such non-smoothness causes an interior layer in the solution. For both classes of problems, a decomposition of the solution is constructed, and pointwise bounds on the partial derivatives of its components are established, showing the location and width of the layer regions. The numerical approximation is based on an upwind finite difference scheme defined on a layer-adapted Shishkin mesh condensing in the layer regions. Parameter-uniform error bounds in the maximum norm are derived, showing that the scheme is robust (or uniformly convergent) with respect to the diffusion parameter. The error bounds prove that the method is almost first-order uniformly convergent and numerical results are presented to illustrate the performance of both numerical methods. The technical details of this work can be found in [1].

Palabras clave: Convection-diffusion ; Interior layer; Shishkin mesh.

Referencias

[1] J.L. Gracia, E. O'Riordan (2025). Singularly perturbed convection-diffusion elliptic problems with a non-smooth forcing term. *Comp. and Maths. with Aplls.*, 196, 458–476.

[2] J.J.H. Miller, E. O'Riordan, G.I. Shishkin, (2012) *Fitted Numerical Methods For Singular Perturbation Problems Error Estimates in the Maximum Norm for Linear Problems in One and Two Dimensions.* Revised Edition, World Scientific Publishing.

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