Generalized RKBS and the Design of Abstract Neural Networks: A Functional Analytic Approach

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Resumen. In this talk, I will present a generalization of Reproducing Kernel Banach Spaces (RKBS), which we call *Generalized Reproducing Kernel Banach Spaces* (GRKBS). The starting point is a recent result [1] showing that classical fully connected neural networks can be interpreted as finite-dimensional subspaces of an RKBS. The proposed generalization allows us to work with function spaces whose values lie in Banach spaces, and, on this basis, to construct *Abstract Neural Networks* (AbsNN) as compositions of GRKBS. This framework provides a natural way to model neural architectures that go beyond traditional machine learning paradigms, incorporating physically informed structures governed by differential equations. In the talk, I will present the unified definition of GRKBS, structural uniqueness results, and the existence of sparse minimizers for the associated abstract training problem. Finally, I will discuss how this framework bridges functional analysis and the design of new neural architectures, with potential applications in approximation theory and mathematical modeling.

Palabras clave: generalized RKBS; abstract training problem; abstract neural structures; sparsity solutions

Referencias

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