

Equisingularity of inner Khovanskii non-degenerate deformations

EDER L. SANCHEZ QUICENO^{1,2}, JULIAN ESPINEL LEAL

¹Departamento de Matemática, Universidade Federal de São Carlos

² Instituto Universitario de Matemática Pura y Aplicada, Universitat Politècnica de València

edersanchez@ufscar.br

Abstract. We present results from [2] concerning the topological equisingularity of singularities arising from real polynomial maps $f : \mathbb{R}^n \rightarrow \mathbb{R}^p$, with $f(0) = 0$, under one-parameter deformation $F(\mathbf{x}, \varepsilon) = f(\mathbf{x}) + \theta(\mathbf{x}, \varepsilon)$, $F(0, \varepsilon) = 0$. The work begins by introducing the notion of *inner Khovanskii non-degeneracy (IKND)*, a condition that generalizes a previous non-degeneracy criterion for complex polynomial functions [5]. We prove that the IKND condition ensures the link of the singularity of f at the origin, $L_f := f^{-1}(0) \cap S_r^{n-1}$, is smooth and well-defined for all small enough radii $r > 0$. Focusing on one-parameter deformations $F(\mathbf{x}, \varepsilon)$, we establish two sets of sufficient conditions: one to guarantee that the topological type of $F_\varepsilon := F(\cdot, \varepsilon)$ remains constant along the deformation (*topological triviality*), and a second set of conditions to ensure the constancy of the isotopy-type of L_{F_ε} (*link-constancy*). Finally, inspired by earlier works [1, 3, 4], we extend these definitions and results to the domain of mixed polynomial maps $f = (f^1, \dots, f^p) : \mathbb{C}^n \rightarrow \mathbb{C}^p$, with $f(0) = 0$, which are complex valued maps such that each f^j is a complex polynomial in the variables $\mathbf{z} = (z_1, \dots, z_n)$ and their conjugates $\bar{\mathbf{z}} = (\bar{z}_1, \dots, \bar{z}_n)$.

Keywords: Deformations; Inner non-degeneracies; Link-constancy; Topological triviality; Real maps.

References

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