Darboux's theorem in p-adic symplectic geometry

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Abstract. Darboux's Theorem [2] is an important result in symplectic geometry, which states that symplectic manifolds have no local invariants other than the dimension. This is usually proved using Moser's Path Method [3].
We derive an analog of Moser's Path Method for p-adic analytic manifolds and use it to prove a p-adic analog of Darboux's Theorem. Hence, in the p-adic category there are also no local symplectic invariants other than the dimension: locally near any point, a $2n$-dimensional p-adic analytic symplectic manifold is symplectomorphic to $((\mathbb{Q}_p)^{2n}, \sum_{i=1}^{n} \mathrm{d}x_i \wedge \mathrm{d}y_i)$.
Using the p-adic Darboux's Theorem as a stepping stone we derive two striking consequences: global Darboux's coordinates always exist for any second countable p-adic analytic symplectic manifold and that two second countable p-adic analytic symplectic manifolds with the same volume and which are both compact or both noncompact must be symplectorphic. This is a symplectic version of a classical theorem of Serre in p-adic analytic geometry [4], which states that every compact p-adic analytic manifold is diffeomorphic to a union of at most $p-1$ balls.
A full version of this abstract can be found in [1].

Keywords: Symplectic geometry; p-adic numbers; Darboux's Theorem; Serre's Theorem.

References

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