Sobolev orthogonal polynomials involving an exponential weight

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Resumen. In this work, we consider the orthogonal polynomials $\{q_n\}_{n\geq 0}$ with respect to a Sobolev inner product involving the Freud weight $\exp(-x^4)$, given by [1]

$$(p,q)_S = \int_{-\infty}^{\infty} p(x)q(x)\ \exp(-x^4)dx + \lambda \int_{-\infty}^{\infty} p'(x)q'(x)\exp(-x^4)dx, \quad \lambda > 0.$$

Our first objective is to study the local asymptotics known as Mehler–Heine asymptotics for this family of polynomials. Moreover, this result together with the well-known Hurwitz's Theorem allows us to establish the asymptotic behavior of the zeros of these polynomials. Motivated by this consequence, we also address the problem of computing the zeros of the polynomials $\{q_n\}_{n\geq 0}$. With that purpose, we construct a generalized eigenvalue problem from the five-term recurrence relation satisfied by these polynomials, so that the computation of the zeros is equivalent to computing the generalized eigenvalues. We also present numerical experiments illustrating the computation of the zeros for different values of λ. Finally, we deduce the fourth-order differential equation satisfied by these polynomials.

Palabras Clave: Sobolev orthogonal polynomials; Mehler–Heine aymptotics; Zeros; Computation.

Referencias

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